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TECHNICAL REPORT NO. S 1001

SIMULATION WITH MARKOV TRANSITION MATRIX MODEL OF A REQUISITION PROCESSING SYSTEM

OPERATIONS RESEARCH BRANCH
OPERATIONS IMPROVEMENT DIVISION

May 1969

by Irwin F. Goodman

TACOM

MANAGEMENT AND DATA SYSTEMS DIRECTORATE
U S ARMY TANK AUTOMOTIVE COMMAND Warren Michigan

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SIMULATION WITH MARKOV
TRANSITION MATRIX MODEL OF
A REQUISITION PROCESSING SYSTEM

IRWIN F. GOODMAN

ABSTRACT

A cursory review of the literature relating to the application of the Markov Transition Probability Matrix for the evaluation and analysis of problems was accomplished. A FORTRAN IV computer time sharing program, based upon the mathematics of Markov transition Matrices, has been developed and documented. The program was initially developed with data based upon a classical random walk problem involving a drunk meandering from corner to corner between his home and a bar. The resulting Markov Model has been applied to a requisitioning system, an essentially equivalent problem. Some analysis results are presented following the application of the computer program to a requisitioning system. The computer program has been written generally enough for application to such other diverse problem areas as charge accounts, tank battles and reliability and maintainability.

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FOREWORD

The author wishes to acknowledge Mr. Larry Pyles, operations research analyst and Mr. Mike Spinelli, management analyst for their assistance during preparation of the graphs and the flow charts.

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TABLE OF CONTENTS

A. Introduction

A cursory review of the literature related to Markov transition probability matrix modeling was accomplished. A short bibliography of related books is included in a later section. Some of the more essential Markov techniques of analysis were organized together and then programed in FORTRAN for solution on a computer time sharing terminal. The purpose of the model is to describe a requisition processing system. An equivalent problem involves the classical random walk problem involving a drunk meandering from corner to corner between his home and a bar. Therefore, initially, the computer program and model were developed and checked out based upon the random walk problem. Then, the model was applied to a requisitioning system.

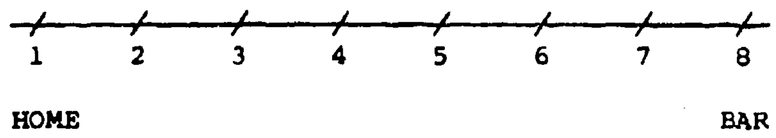
The data used in this report are presented only for the sake of illustrating the theory employed. The findings are consistent with the data presented but therefore not necessarily with the real-world situation.

B. Markov Chains

A Markov Chain is defined as a probabilistic process in which the probability of moving from one state to another state may depend on the present state, but on no other past history. Classically, this process is usually examplified in terms of the "wandering drunk" problem which is an example of a random walk.

"Wandering Drunk" Problem

A long street has eight intersections. A drunk wanders along the street. His home is located at intersection 1 and his favorite bar at intersection number 8.



Intersections 2 - 7 Are Referred To As Street Corners

At each intersection other than his home or the bar, he moves in the direction of the bar with probability 1/4 and in the direction of his home with probability 3/4. He never wanders down a side street. If he arrives at his home or the bar, he remains there. When he remains, we say that the process is "absorbed".

Some of the typical questions that an analysis of this problem would address itself to are as follows:

a. What is the chance that, starting at a given corner, the drunk will end up at his home or at the bar?

b. If the drunk starts at a particular corner, how many blocks, on the average, will the drunk walk before being "absorbed", that is, arrive at his home or the bar?

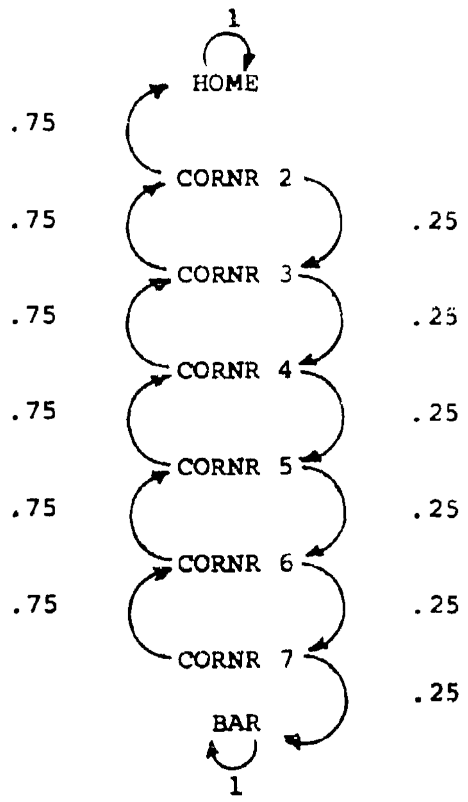
Stated in the Markov language of probability: What are the absorption probabilities, and what is the mean time to absorption?

The transition probability matrix and diagram for this problem are as follows:

TRANSITION PROBABILITY MATRIX

STATE	HOME	BAR	CORNR 2	CORNR 3	CORNR 4	CORNR 5	CORNR 6	CORNR 7
Home	1	0	0	0	0	0	0	0
Bar	0	1	0	0	0	0	0	0
CORNR 2	.75	0	0	.25	0	0	0	0
CORNR 3	0	0	.75	0	.25	0	0	0
CORNR 4	0	0	0	.75	0	.25	0	0
CORNR 5	0	0	0	0	.75	0	.25	0
CORNR 6	0	0	0	0	0	.75	0	.25
CORNR 7	0	.25	0	0	0	0	.75	0

TRANSITION PROBABILITY DIAGRAM



In summary, the home and bar states are referred to as absorbing states and the other states, CORNR 2 thru CORNR 7, are referred to as transient states. In the case of absorption states, once there, the probability of remaining there is one (certainty). On the other hand with regard to the transition states, it is possible to leave as well as enter them.

The transition probability matrix for the "wandering drunk" problem was processed by the MARK1 computer program. The computer print - out is included in a later section. The results are briefly discussed as follows: Assuming the drunk initially can be at any one of the corners 2 thru 7 (away from the home or bar) with equal probability 1/6, then in the long run he will end up at his home with probability .92 and end up at the bar with probability .08. With regard to the typical questions discussed earlier, the results are as follows:

The probability that starting at a given corner, the drunk will end up at his home or the bar are as follows:

STATE	HOME	BAR
CORNR 2	.999	.001
CORNR 3	.996	.004
CORNR 4	.988	.012
CORNR 5	.963	.037
CORNR 6	.889	.111
CORNR 7	.667	.333

If the drunk starts at a given corner, the quantity of blocks, on the average, that he will walk before arriving at his home or the bar are as follows:

INITIAL STATE	AVERAGE QUANTITY OF BLOCKS TO HOME OR BAR
CORNR 2	2.0
CORNR 3	3.9
CORNR 4	5.8
CORNR 5	7.5
CORNR 6	8.5
CORNR 7	7.3

C. "A Requisition Processing System" Problem

Markov Transition Probability Matrix reasoning and techniques of analysis which were briefly discussed above in terms of a classical random walk problem have been applied to a requisition processing system. The system has been greatly simplified here for the sake of presentation of ideas and the mathematical modeling. The system is primarily automatic, consisting of a high speed automatic data processing computer system and the logic of a highly sophisticated inventory control system which together process the transactions. A requisition represents a demand on the system by a customer for materiel. This type of transaction (requisition) and the many other transactions necessary to keep the records current and decisions reliable are processed by the computer inventory control system.

A successful attempt has been made to mathematically model some of the aspects of such a system. To begin with, only the requisition type transactions were studied. Such a transaction was considered to be in the following states:

Actions completed:

MRO - Materiel Release Order

PASORD - Passing Order

REJCUS - Reject to Customer

BACKOR - Back-Order

Actions to be completed:

FRSPAS - First-Pass

SUBPAS - Subsequent - Pass

COMPLT - Completed action is defined as any one of the above four states (MRO, PASORD, REJCUS, BACKOR)

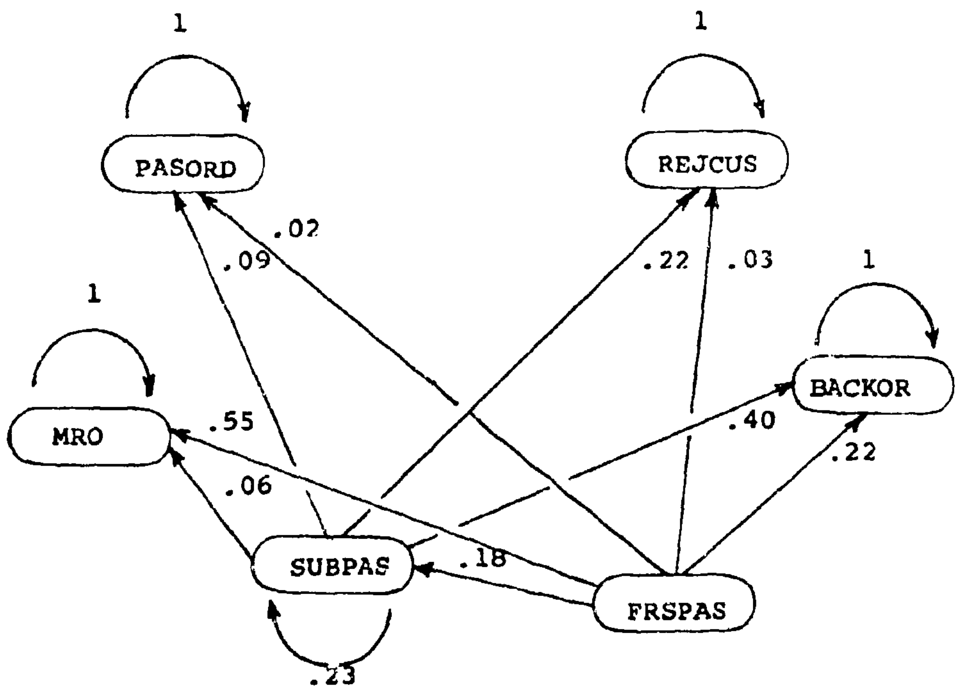
Two versions were modeled, one for all the above states, four absorbing and two transient, and the other one containing only one absorbing state (in place of the four absorbing states) and two transient states (same two transient states). The models are referred to as Model A and Model B. The states considered for each model are shown below:

MODEL A	MODEL B
<u>Absorption States:</u>	<u>Absorption States:</u>
MRO	COMPLT
PASORD	
REJCUS	
BACKOR	
<u>Transient States:</u>	<u>Trainsient States:</u>
FRSPAS	FRSPAS
SUBPAS	SUBPAS

The Model B version was defined with less states to facilitate a more in-depth variation of parameter study. The results of the variation of parameter study are presented in a later section. The transition probability matrices and diagrams for the Model A and Model B versions of the requisition processing system are as follows:

MODEL A

TRANSITION PROBABILITY DIAGRAM

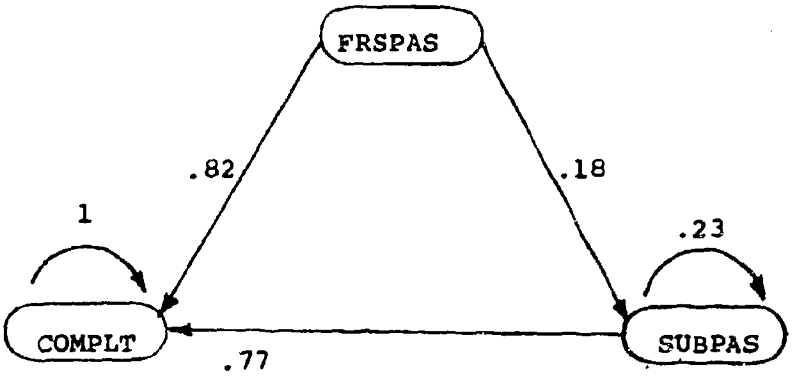


TRANSITION PROBABILITY MATRIX

STATE:	MRO	PASORD	REJCUS	BACKOR	FRSPAS	SUBPAS
MRO	1	0	0	0	0	0
PASORD	0	1	0	0	0	0
REJCUS	0	0	1	0	0	0
BACKOR	0	0	0	1	0	0
FRSPAS	.55	.02	.03	.22	0	.18
SUBPAS	.06	.09	.22	.40	0	.23

MODEL B

TRANSITION PROBABILITY DIAGRAM



TRANSITION PROBABILITY MATRIX

	COMPLT	FRSPAS	SUBPAS
COMPLT	1	0	0
FRSPAS	.82	0	.18
SUBPAS	.77	0	.23

The earlier questions regarding the "drunk" in the classical random walk problem can now be restated in the context of the requisition processing system as follows:

a. Given the transaction is in one of the transient states (first pass or subsequent pass), what is the probability the transaction will end up as a "MRO"?; as a passing order?; as a reject to customer?; as a backorder?

b. Given the transaction is in one of the transient states (first pass or subsequent pass), how many times, on the average, will the transaction be in the subsequent pass state prior to ending up as a MRO?; as a passing order?; as a reject to customer?; as a backorder?

D. Some Results with Model A

1. The Model A of the requisition processing system was processed by the MARK1 computer program. The computer print-out is included in a later section. Some of the results are as follows:

2. The expected number of times of remaining in a transient state prior to being absorbed are obtained from the fundamental matrix. The results are as follows:

a. A first pass (FRSPAS) transaction can be expected to be in a first pass state only once prior to being absorbed. This, agrees as it should with the "common sense" of the system.

b. A subsequent pass (SUBPAS) transaction can be expected to be in a first pass state zero times prior to being absorbed. This also agrees as it should with the "common sense" of the system by implying that once a transaction is a subsequent pass transaction it cannot pass through the first pass state.

c. A first pass (FRSPAS) transaction can be expected to be in a subsequent pass state about .23 times before being absorbed.

d. A subsequent pass transaction can be expected to be in a subsequent pass state 1.30 times prior to being absorbed.

3. The mean number of cycles until absorbtion is obtained from the T Matrix.

a. The mean number of cycles until completion for a first pass transaction is 1.23.

b. The mean number of cycles until completion for a subsequent pass transaction is 1.30.

4. The probability that a transaction is completed given it was initially in a particular state is obtained from the U Matrix.

a. The probability is .56 that a first pass transaction will become an MRO and .08 that a subsequent pass transaction will become an MRO.

b. The probability is .04 that a first pass transaction will become a passing order and .12 that a subsequent pass transaction will become a passing order.

c. The probability is .08 that a first pass transaction will become a reject to customer and .29 that a subsequent pass transaction will become a reject to customer.

d. The probability is .31 that a first pass transaction will go on back-order and .52 that a subsequent pass transaction will go on back-order.

5. The number of cycles required to reduce the fraction of subsequent pass transactions to 1% or less is obtained from computation of the AM(K) vector.

a. Assuming initially that 20% of the transactions to be processed are subsequent pass, it can be expected to take three (3) cycles for the fraction of subsequent pass transactions to drop to 1% or less. In two (2) cycles the fraction of subsequent pass transactions can be expected to drop to about 4%.

b. The same percentages as above essentially hold even if initially 100% of the transactions to be processed are subsequent pass transactions.

E. A Variation of Parameter Study on Model B.

1. Initially, Model B was run with transition Matrix values as depicted in the earlier section showing the transition matrix diagram. The computer print-out is included in a later section.

2. A variation of parameter study was conducted on Model B. The purpose of the study is to develop charts and graphs depicting relationships between the input and output of Model B over a complete range in values of particular parameters. The parameters involved, their possible range in variation, and the parametric values studied are shown in the following table:

Parameter	Possible Range	Parametric Values Studied
First Pass to Subsequent Pass Transition Probability	0.0 to 1.0	.01, .05, .10, .15, .20, .25, .50, .75, .99, .999
Subsequent Pass to Subsequent Pass Transition Probability	0.0 to 1.0	.05, .10, .25, .50

INPUT: Model B was studied for three initial vector conditions.

(1) (0, 1, 0): 100% first pass transactions and 0% subsequent pass transactions.

(2) (0, .80, .20): 80% first pass transaction and 20% subsequent pass transactions.

(3) (0, 0, 1): 0% first pass transactions, and 100% subsequent pass transactions.

OUTPUT: Model B was studied for the following Outputs:

(1) The model was studied for the number of cycles required to yield an output vector such that the percent of subsequent pass transactions (incompleted transactions) is 1% or less. This is equivalent to the number of cycles required to yield an output vector such that the percent of completed transactions is 99% or more.

(2) Also, the average quantity of cycles expected for a first pass transaction and a subsequent pass transaction to be completed were studied.

3. The results are presented on the following charts. Chart E-1 summarizes the results obtained for 100% first-pass input and also for the mixed input (80% first-pass and 20% subsequent pass). The solid curves represent the 100%

first-pass input and the broken curves represent the mixed input.

The other input condition, 100% subsequent pass, is shown on Chart E-2.

The average quantity of cycles expected for a first-pass transaction to be completed is presented on Chart E-3. Chart E-4 shows the average quantity of cycles expected for a subsequent-pass transaction to be completed.

The basic data for preparing Charts E-1 and E-2 is presented in a tabular format on Charts E-5 thru E-8.

Discussion and interpretation of the charts and other findings is being withheld at this time pending further study of the model.

CHART E-1

QUANTITY OF CYCLES REQUIRED SUCH THAT THE PERCENT OF OUTPUT TRANSACTIONS FOR SUBSEQUENT-PASS (INCOMPLETE) IS 1% OR LESS

INITIAL CONDITIONS:

100% FIRST PASS INPUT (0, 1, 0) AND MIXED INPUT (0, .80, .20)

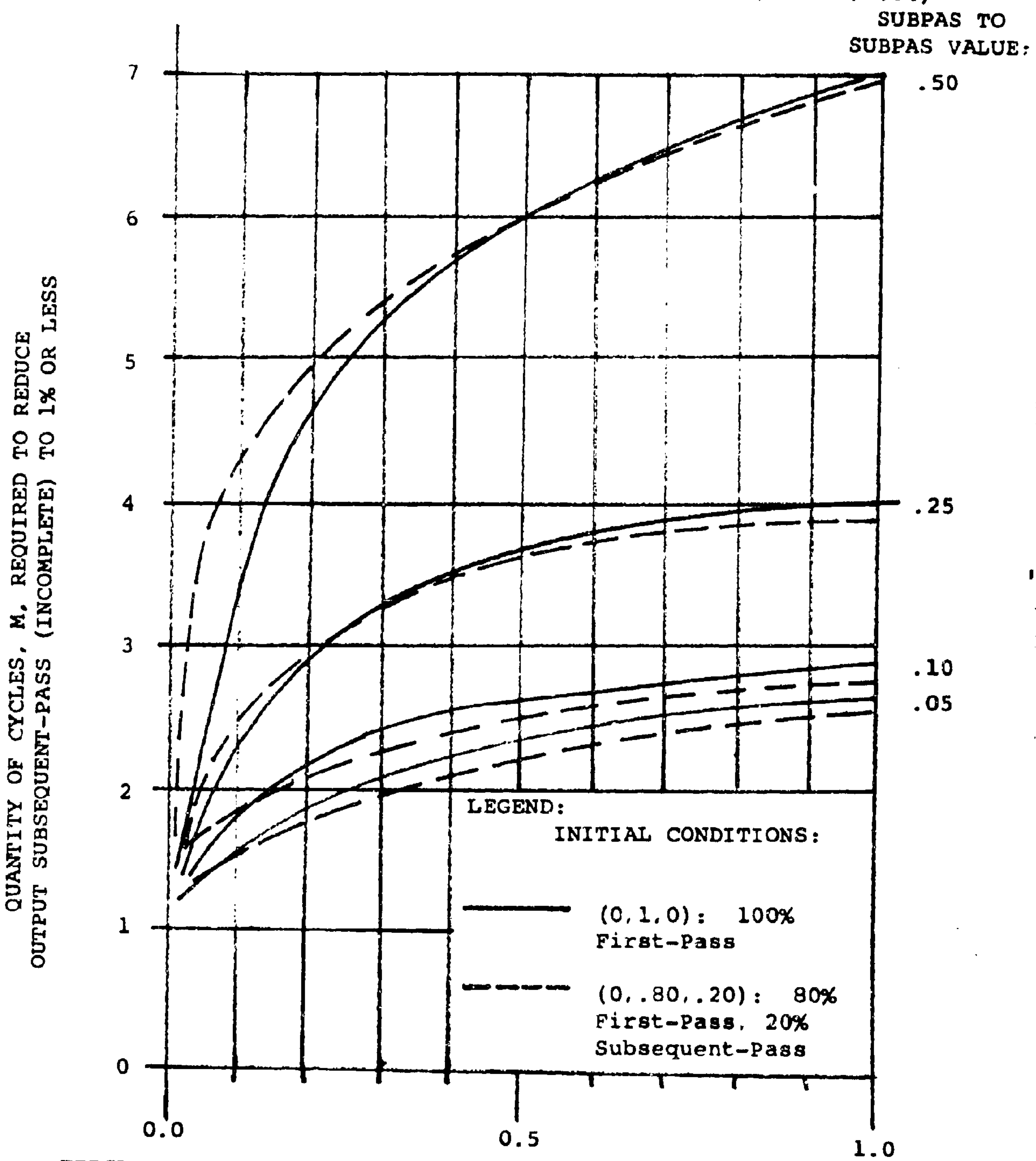


CHART E-2

QUANTITY OF CYCLES REQUIRED SUCH THAT THE PERCENT OF OUTPUT TRANSACTIONS FOR SUBSEQUENT-PASS (INCOMPLETE) IS 1% OR LESS

INITIAL CONDITIONS:

100% SUBSEQUENT-PASS INPUT (0, 0, 1)

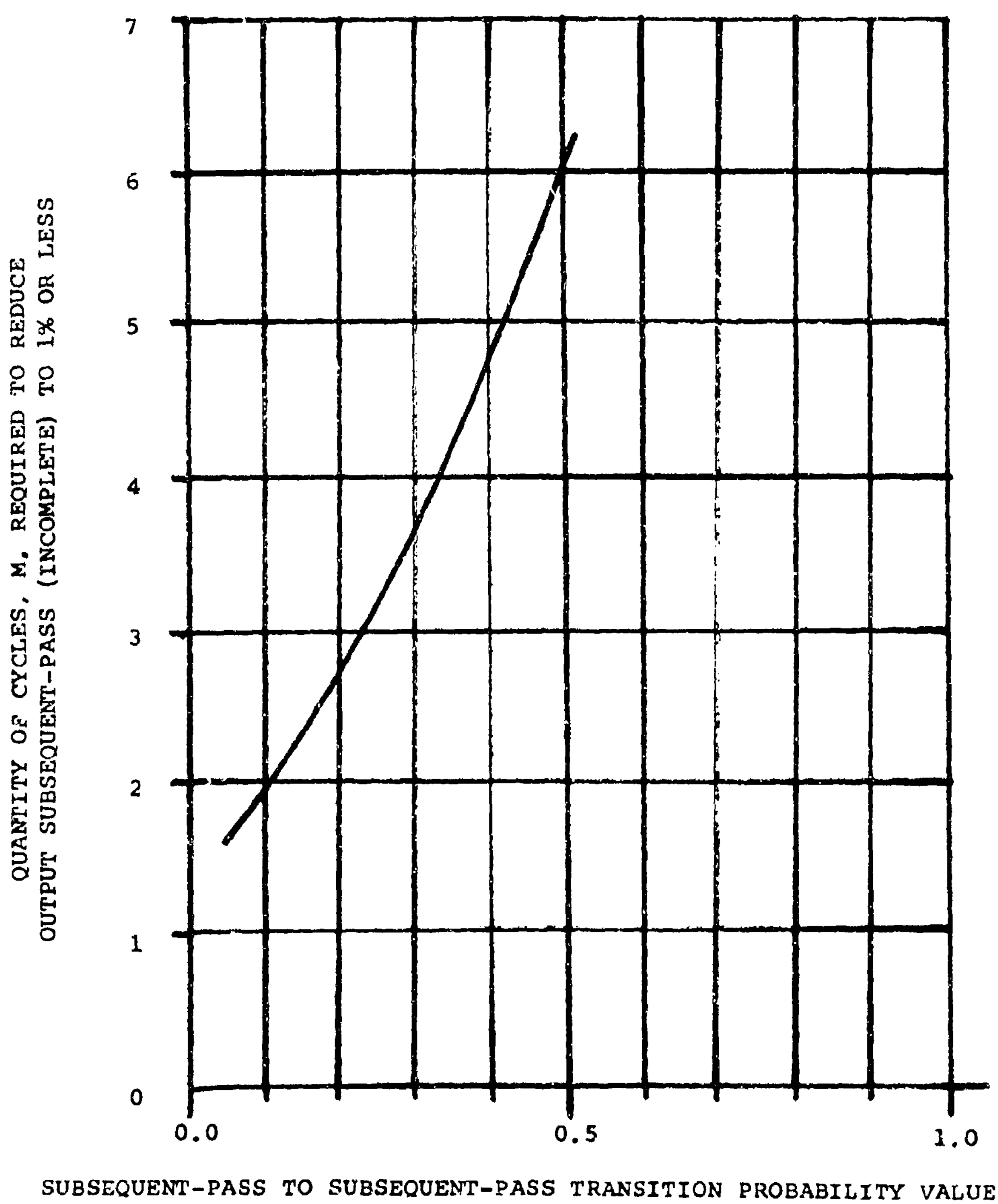


CHART E-3

AVERAGE QUANTITY OF CYCLES EXPECTED FOR A
FIRST-PASS TRANSACTION TO BE COMPLETED

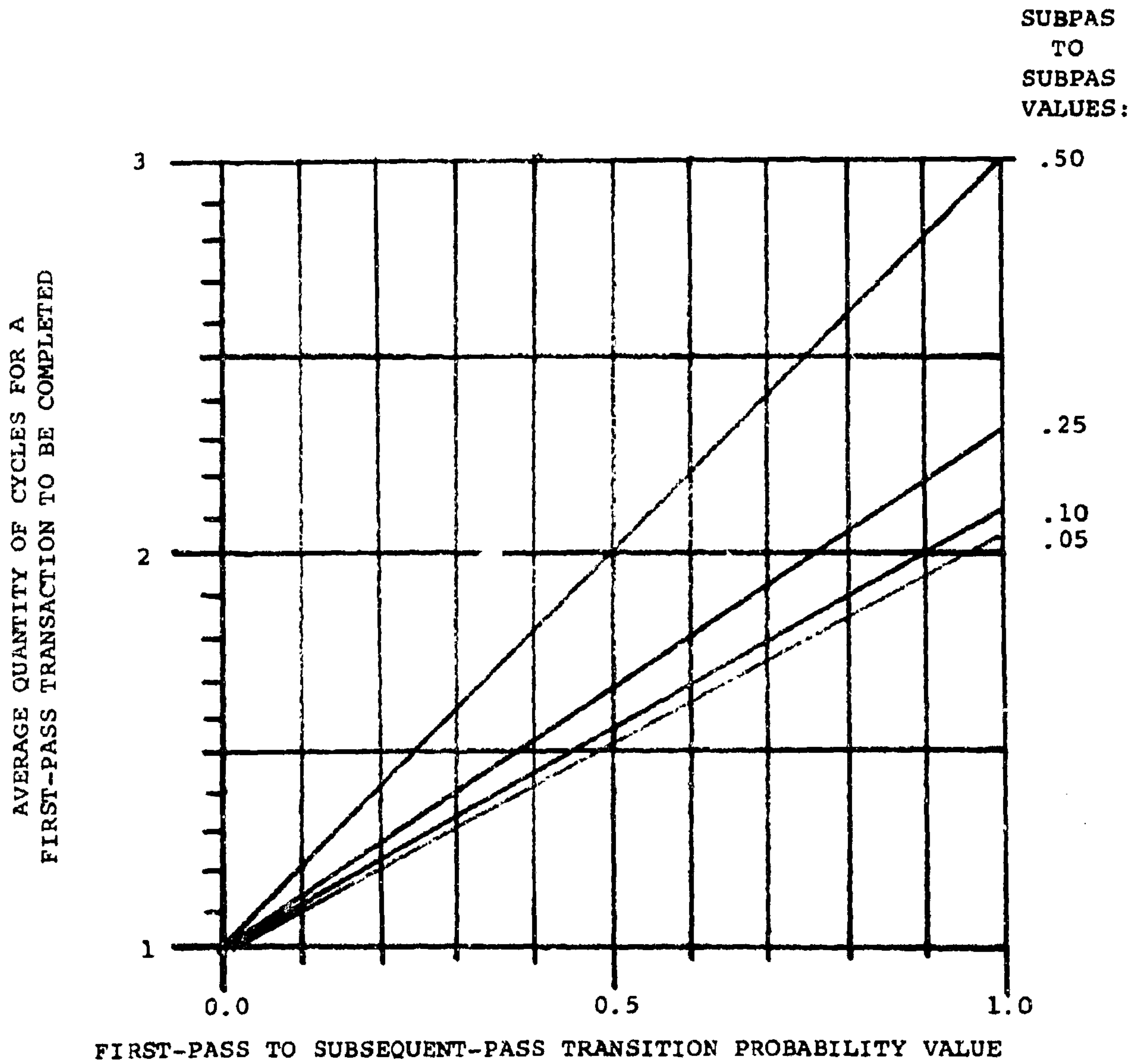


CHART E-4

AVERAGE QUANTITY OF CYCLES EXPECTED FOR A SUBSEQUENT-PASS TRANSACTION TO BE COMPLETED

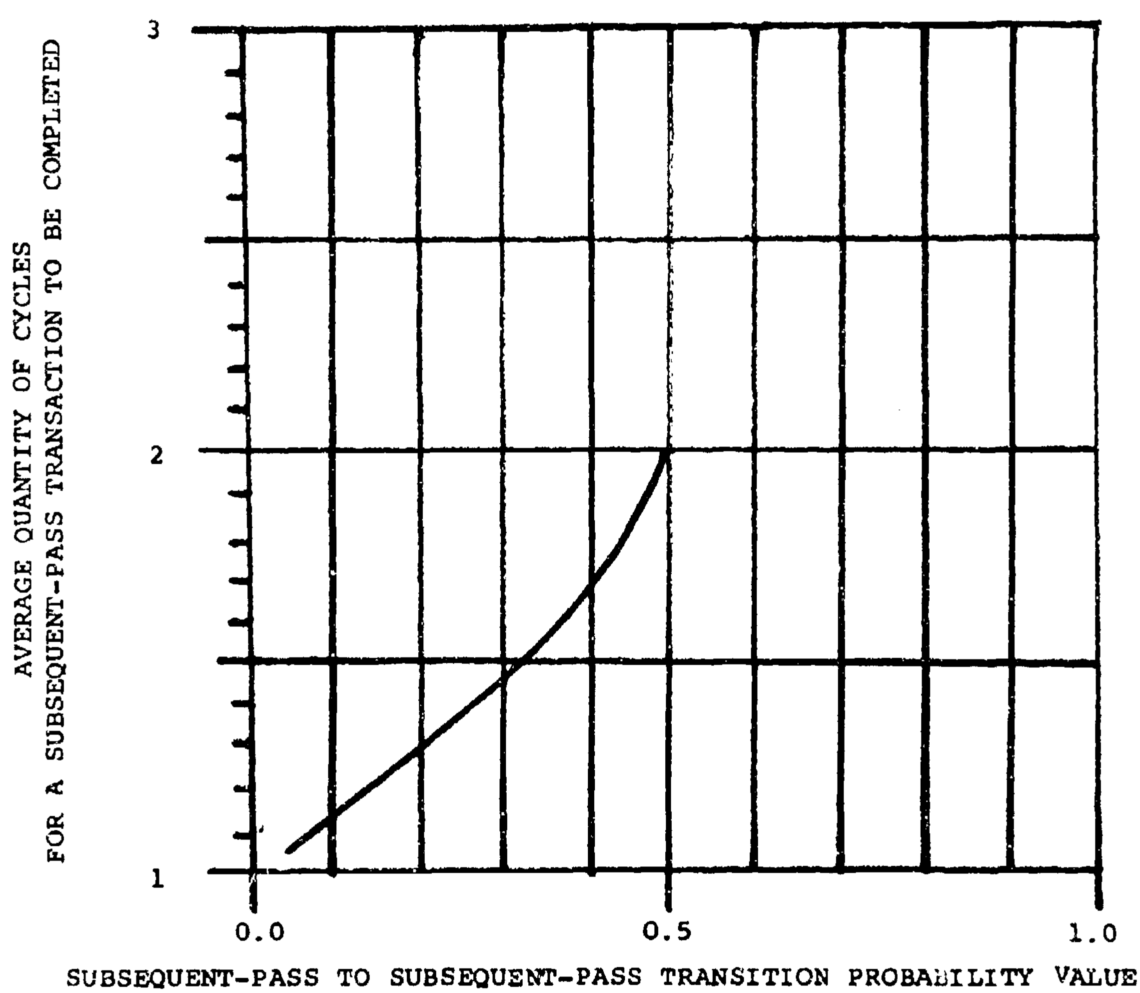


CHART E-5

TABULATION OF BASIC DATA
SUB-PASS TO SUB-PASS TRANSITION PROBABILITY VALUE=.05

QTY CYCLES (STEPS)	FIRST PASS TO SUBSEQUENT PASS TRANSITION PROBABILITY VALUES									
	.999	.99	.75	.50	.25	.20	.15	.10	.05	.01
	INITIAL CONDITION *: (0, 1, 0)									
1	.999	.990	.750	.500	.250	.200	.150	.100	.050	.010
2	.050	.049	.037	.025	.012	.010	.007	.005	.002	.000
3	.002	.002	.002	.001	.001					
4										
5										
6										
7										
8										
	INITIAL CONDITION *: (0, .80, .20)									
1	.810	.810	.610	.410	.210	.170	.130	.090	.050	.002
2	.040	.040	.030	.020	.010	.008	.007	.004	.002	.000
3	.002	.002	.002	.001	.001					
4										
5										
6										
7										
8										
	INITIAL CONDITION *: (0, 0, 1)									
1	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050
2	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002
3										
4										
5										
6										
7										
8										

*INITIAL CONDITION (% Computed., % FIRST PASS, % SUBSEQUENT PASS)

CHART E-5

TABULATION OF BASIC DATA
SUB-PASS TO SUB-PASS TRANSITION PROBABILITY VALUE= .10

QTY CYCLES (STEPS)	FIRST PASS TO SUBSEQUENT PASS TRANSITION PROBABILITY VALUES									
	.999	.99	.75	.50	.25	.20	.15	.10	.05	.01
	INITIAL CONDITION *: (0, 1, 0)									
1	.999	.990	.750	.500	.250	.200	.150	.100	.050	.010
2	.100	.100	.075	.050	.025	.020	.015	.010	.005	.001
3	.010	.010	.007	.005	.002	.002				
4										
5										
6										
7										
8										
	INITIAL CONDITION *: (0, .80, .20)									
1	.820	.820	.620	.420	.220	.180	.140	.100	.060	.030
2	.082	.082	.062	.042	.022	.018	.014	.010	.006	.003
3	.008	.008	.006	.004	.002					
4										
5										
6										
7										
8										
	INITIAL CONDITION *: (0, 0, 1)									
1	.100	.100	.100	.100	.100	.100	.100	.100	.100	.100
2	.010	.010	.010	.010	.010	.010	.010	.010	.010	.010
3										
4										
5										
6										
7										
8										

*INITIAL CONDITION (% Computed., % FIRST PASS, % SUBSEQUENT PASS)

CHART E-7

TABULATION OF BASIC DATA
SUB-PASS TO SUB-PASS TRANSITION PROBABILITY VALUE=.25

QTY CYCLES (STEPS)	FIRST PASS TO SUBSEQUENT PASS TRANSITION PROBABILITY VALUES									
	.999	.99	.75	.50	.25	.20	.15	.10	.05	.01
	INITIAL CONDITION *: (0, 1, 0)									
1	.999	.990	.750	.500	.250	.200	.150	.100	.050	.010
2	.250	.250	.187	.125	.062	.050	.037	.025	012	002
3	.062	.062	.047	.031	.016	.012	.009	.006	.003	.001
4	.016	.015	.012	.008	.004					
5	.004	.003								
6										
7										
8										
	INITIAL CONDITION *: (0, .80, .20)									
1	.850	.850	.650	.450	.250	.210	.170	.130	.090	.060
2	.212	.210	.162	.112	.062	.050	.042	.032	.022	.014
3	.053	.053	.041	.028	.016	.013	.010	.008	.006	.004
4	.014	.012	.010	.007	.004					
5	.003									
6										
7										
8										
	INITIAL CONDITION *: (0, 0, 1)									
1	.250	.250	.250	.250	.250	.250	.250	.250	.250	.250
2	.062	.062	.062	.062	.062	.062	.062	.062	062	.062
3	.016	.016	.016	.016	.016	.016	.016	.016	.016	.016
4	.004	.004	.004	.004	.004	.004	.004	.004	.004	.004
5										
6										
7										
8										

*INITIAL CONDITION (% Computed., % FIRST PASS, % SUBSEQUENT PASS)

CHART E-8

TABULATION OF BASIC DATA
SUB-PASS TO SUB-PASS TRANSITION PROBABILITY VALUE= .50

QTY CYCLES (STEPS)	FIRST PASS TO SUBSEQUENT PASS TRANSITION PROBABILITY VALUES									
	.999	.99	.75	.50	.25	.20	.15	.10	.05	.01
	INITIAL CONDITION *: (0, 1, 0)									
1	.999	.990	.750	.500	.250	.200	.150	.100	.050	.010
2	.499	.499	.375	.250	.125	.100	.075	.050	.025	.005
3	.250	.250	.187	.125	.062	.050	.038	.025	.012	.002
4	.125	.125	.094	.062	.031	.025	.019	.012	.006	.001
5	.062	.062	.047	.031	.016	.012	.009	.006	.003	.001
6	.031	.031	.023	.016	.008	.006	.005	.003	.002	
7	.016	.015	.012	.008	.004	.003	.002	.001	.001	
8	.008	.008								
	INITIAL CONDITION *: (0, .80, .20)									
1	.892	.892	.700	.500	.300	.260	.220	.180	.140	.110
2	.450	.450	.350	.250	.150	.130	.110	.090	.070	.054
3	.225	.225	.175	.125	.075	.065	.055	.045	.035	.027
4	.112	.112	.087	.062	.037	.032	.028	.022	.017	.013
5	.056	.056	.044	.031	.019	.016	.013	.011	.009	.007
6	.028	.028	.022	.016	.010	.008	.007	.006	.005	.003
7	.014	.014	.011	.008	.005	.004	.003	.003	.001	
8	.007	.007								
	INITIAL CONDITION *: (0, 0, 1)									
1	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500
2	.250	.250	.250	.250	.250	.250	.250	.250	.250	.250
3	.125	.125	.125	.125	.125	.125	.125	.125	.125	.125
4	.062	.062	.062	.062	.062	.062	.062	.062	.062	.062
5	.031	.031	.031	.031	.031	.031	.031	.031	.031	.031
6	.016	.016	.016	.016	.016	.016	.016	.016	.016	.016
7	.008	.008	.008	.008	.008	.008	.008	.008	.008	.008
8										

*INITIAL CONDITION (% Computed., % FIRST PASS, % SUBSEQUENT PASS)

F. Mathematical Terminology and Formulae

1. <u>Transition Probability Matrix</u>: A square array of numbers all of whose entries for each row add up to one.

1.1 <u>Example</u>:

		NEXT STATE							
	STATE	HOME	2	3	4	5	6	7	BAR
	HOME	1	0	0	0	0	0	0	0
	2	3/4	0	1/4	0	0	0	0	0
PRESENT	3	0	3/4	0	1/4	0	0	0	0
STATE	4	0	0	3/4	0	1/4	0	0	0
	5	0	0	0	3/4	0	1/4	0	0
	6	0	0	0	0	3/4	0	1/4	0
	7	0	0	0	0	0	3/4	0	1/4
	BAR	0	0	0	0	0	0	0	1

1.2 <u>Explanation</u>: The entries for each particular row represent the probabilities of an item going to the corresponding column state given it was initially in the state corresponding to the particular row. i.e., The probability of going from state 4 to state 2 is 0, to state 3 is .750, and to state 5 is .250.

2. <u>Absorbing States</u>: Row probability vectors in the transition probability matrix having "1s" on the diagonal of the matrix and all the other entries are "0s" are referred to as absorbing states, i.e., row "home" and row "bar" are absorbing states.

3. <u>Transient States (non-absorbing)</u>: Row probability vectors in the transition probability matrix not having "1s" on the diagonal of the matrix are referred to as transient states. i.e., rows "2", "3", "4", and "5" are transient states.

4. <u>Canonical Form of Transition Probability Matrix</u>: When in the transition probability matrix, all the absorbing states (rows) are grouped together at the top of the matrix with all the "1s" composing an identity matrix and all of the transient states (rows) are together at the bottom of the matrix, then, the transition probability matrix is said to be in canonical form.

4.1 <u>Example</u>:

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
HØME	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BAR	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
CØRNR2	.750	0.000	0.000	.250	0.000	0.000	0.000	0.000
CØRNR3	0.000	0.000	.750	0.000	.250	0.000	0.000	0.000
CØRNR4	0.000	0.000	0.000	.750	0.000	.250	0.000	0.000
CØRNR5	0.000	0.000	0.000	0.000	.750	0.000	.250	0.000
CØRNR6	0.000	0.000	0.000	0.000	0.000	.750	0.000	.250
CØRNR7	0.000	.250	0.000	0.000	0.000	0.000	.750	0.000

5. <u>Partioning Canonical Matrix</u>: The canonical matrix is subdivided as follows:

$$C = \left(\begin{array}{c|c} CI & C\emptyset \\ \hline CR & CQ \end{array}\right)$$

5.1 CI: Identity Matrix: The number of rows equals the number of columns equals the number of absorbing states.

5.11 Example:

CI =

STATE	HØME	BAR
HØME	1.000	0.000
BAR	0.000	1.000

5.2 CØ: Zero Matrix: A matrix containing all zeros and having the number of rows equal to the number of absorbing states and the number of columns equal to the number of transient states.

5.21 Example:

$$C\emptyset = \begin{pmatrix} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

5.3 CR: Matrix of Transient to Absorbing Probabilities: The entries represent the probability of going from transient state to absorbing state. The number of rows equals the number of transient states and the number of columns equals the number of absorbing states.

5.31 Example:

CR=

STATE	HØME	BAR
CØRNR2	.750	0.000
CØRNR3	0.000	0.000
CØRNR4	0.000	0.000
CØRNR5	0.000	0.000
CØRNR6	0.000	0.000
CØRNR7	0.000	.250

5.4 CQ: Matrix of Transient to Transient Probabilities:

The entries represent the probability of going from transient state to transient state. The number of rows equals the number of columns equals the number of transient states.

5.41 Example:

CQ =

STATE	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
CØRNR2	0.000	.250	0.000	0.000	0.000	0.000
CØRNR3	.750	0.000	.250	0.000	0.000	0.000
CØRNR4	0.000	.750	0.000	.250	0.000	0.000
CØRNR5	0.000	0.000	.750	0.000	.250	0.000
CØRNR6	0.000	0.000	0.000	.750	0.000	.250
CØRNR7	0.000	0.000	0.000	0.000	.750	0.000

6. FN: Fundamental Matrix: Each entry, FN (I,J), is the expected number of times in state J (column) before being absorbed, given that the present state is I (row). The number of rows equals the number of columns equals the number of transient states.

$$FN = (NI-CQ)^{-1}$$

NOTE: NI is an identity matrix, established for the computation of FN. NI has the number of rows equal to the number of columns equal to the number of transient states.

6.1 Computation of FN: FN is computed, using the following power series approximation:

$$FN - NI + CQ + (CQ)^2 + (CQ)^3 + \ldots.$$

6.2 Example:

FN =

STATE	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
CØRNR2	1.332	.443	.146	.048	.015	.004
CØRNR3	1.328	1.771	.586	.190	.059	.015
CØRNR4	1.317	1.757	1.903	.618	.190	.048
CØRNR5	1.284	1.713	1.855	1.903	.586	.146
CØRNR6	1.186	1.581	1.713	1.757	1.771	.443
CØRNR7	.889	1.186	1.284	1.317	1.328	1.332

7. <u>T: Matrix of Absorption Times</u>: Each entry is the mean time to absorption (number of states passed through, including final state and not including initial state, in order to be absorbed). This matrix is a column vector with the number of rows equal to the number of transient states.

7.1 <u>Computation of T</u>: Each entry of T is equal to the row sum of each row of FN. This is accomplished by establishing a column vector, Ø1 of 1's having the same number of rows equal to the number of transient states.

T = (FN) • (Ø1)

7.2 <u>Example</u>:

T =

STATE	
CØRNR2	1.987
CØRNR3	3.949
CØRNR4	5.833
CØRNR5	7.487
CØRNR6	8.450
CØRNR7	7.337

8. <u>U: Matrix of Absorption Probabilities</u>: Each entry is the probability of being absorbed, given it was initially in a transient state. The number of rows equals the number of transient states, and the number of columns equals the number of absorbing states.

8.1 <u>Computation of U</u>:

U = (FN) • (CR)

8.2 Example:

U =

STATE	HØME	BAR
CØRNR2	.999	.001
CØRNR3	.996	.004
CØRNR4	.988	.012
CØRNR5	.963	.037
CØRNR6	.889	.111
CØRNR7	.667	.333

9. CQMP: Matrix of Transition Probabilities for M Steps: Each entry is the probability of going from state to state. The number of rows equals the number of columns equals the number f absorbing and transient (non-absorbing) states.

9.1 Computation of CQMP:

$$CQMP = (C)^M$$

9.2 Example:

For M = 3

CQMP =

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
HØME	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BAR	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
CØRNR2	.891	0.000	0.000	.094	0.000	.016	0.000	0.000
CØRNR3	.562	0.000	.281	0.000	.141	0.000	.016	0.000
CØRNR4	.422	0.000	0.000	.422	0.000	.141	0.000	.016
CØRNR5	0.000	.016	.422	0.000	.422	0.000	.141	0.000
CØRNR6	0.000	.062	0.000	.422	0.000	.422	0.000	.094
CØRNR7	0.000	.297	0.000	0.000	.422	0.000	.281	0.000

10. <u>A: Initial State Space Probability Row Vector:</u> Initial values for each of the possible states (columns). The number of rows is one, and the number of columns equals the number of absorbing and transient states

10.1 <u>Example</u>:

A = (0, 0, 1/ , 1/6, 1/6, 1/6, 1/6, 1/6)

11. <u>AM: State Space M Steps Later</u>: Each entry is the value for each of the states, M steps later, given initial values A.

11.1 <u>Computation of AM</u>:

AM = (A) • (CQMP)

11.2 Example:

For M=3,

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
	.313	.063	.117	.157	.164	.097	.073	.018

12. <u>Summary</u>: For absorbing Markov chains, the following three questions are usually of interest:

a. What is the probability that the process will end up in a given absorbing state?

Answer: Entries in Matrix U.

b. On the average, how long will it take for the process to be absorbed?

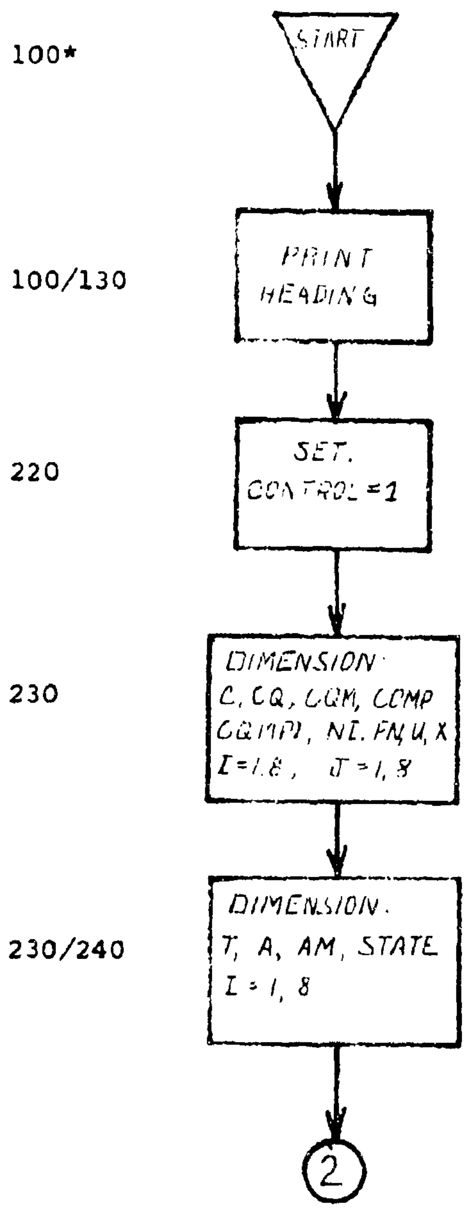
Answer: Entries in Matrix T.

c. On the average, how many times will the process be in each transient (non-absorbing) state?

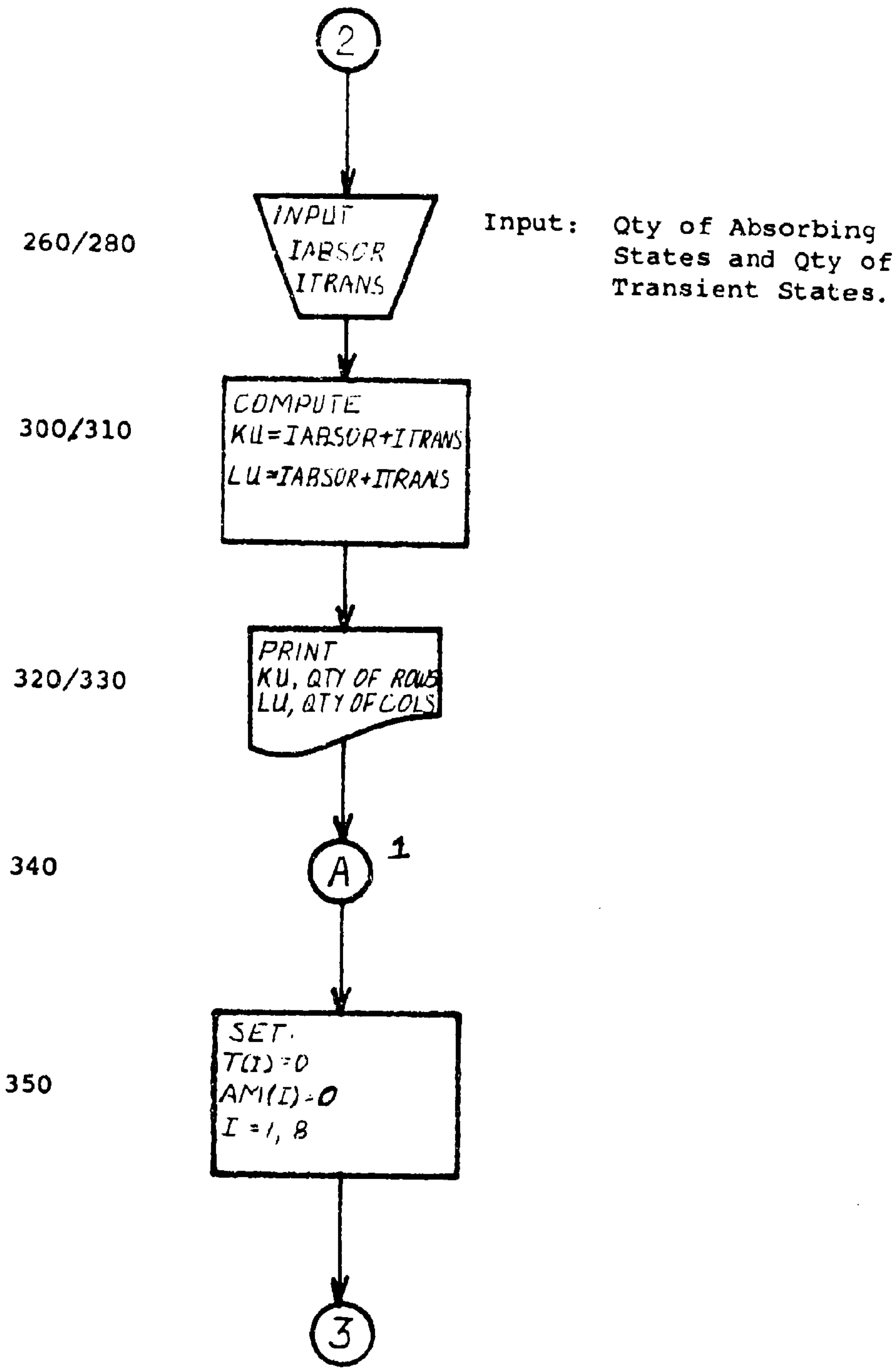
Answer: Entries in Matrix FN.

G. Flow Chart (Fortran Program - MARK1/2/3)
Simulation with Markov
Transition Matrix Model of
A Requisition Processing System

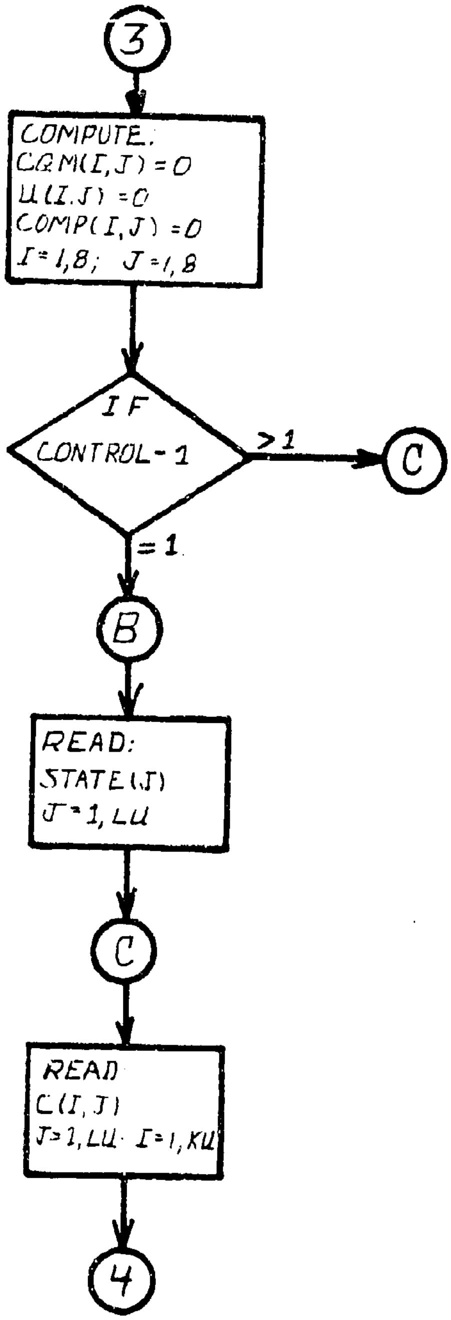
MARK1



*Program Line Numbers


2
260/280
INPUT
IABSOR
ITRANS
Input: Qty of Absorbing States and Qty of Transient States.
300/310
COMPUTE
KU=IABSOR+ITRANS
LU=IABSOR+ITRANS
320/330
PRINT
KU, QTY OF ROWS
LU, QTY OF COLS
340
A
1
350
SET
T(I)=0
AM(I)=0
I=1, 8
3

380
390
400
400
410
420



500

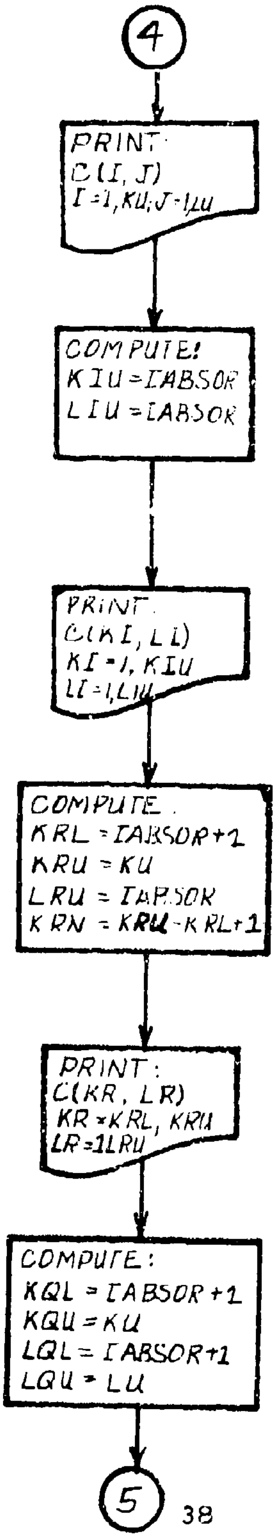
580/590

620

690/720

730

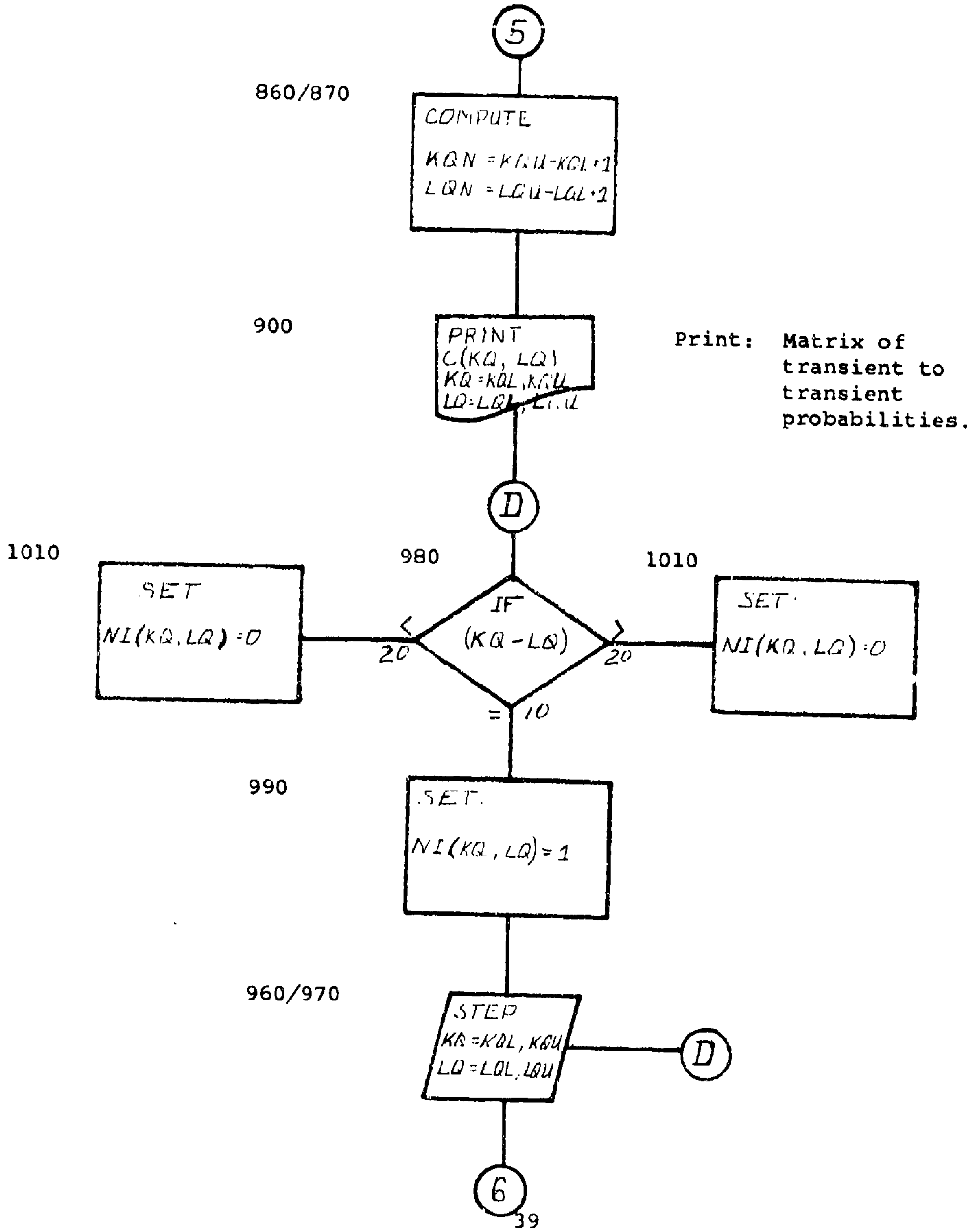
820/850



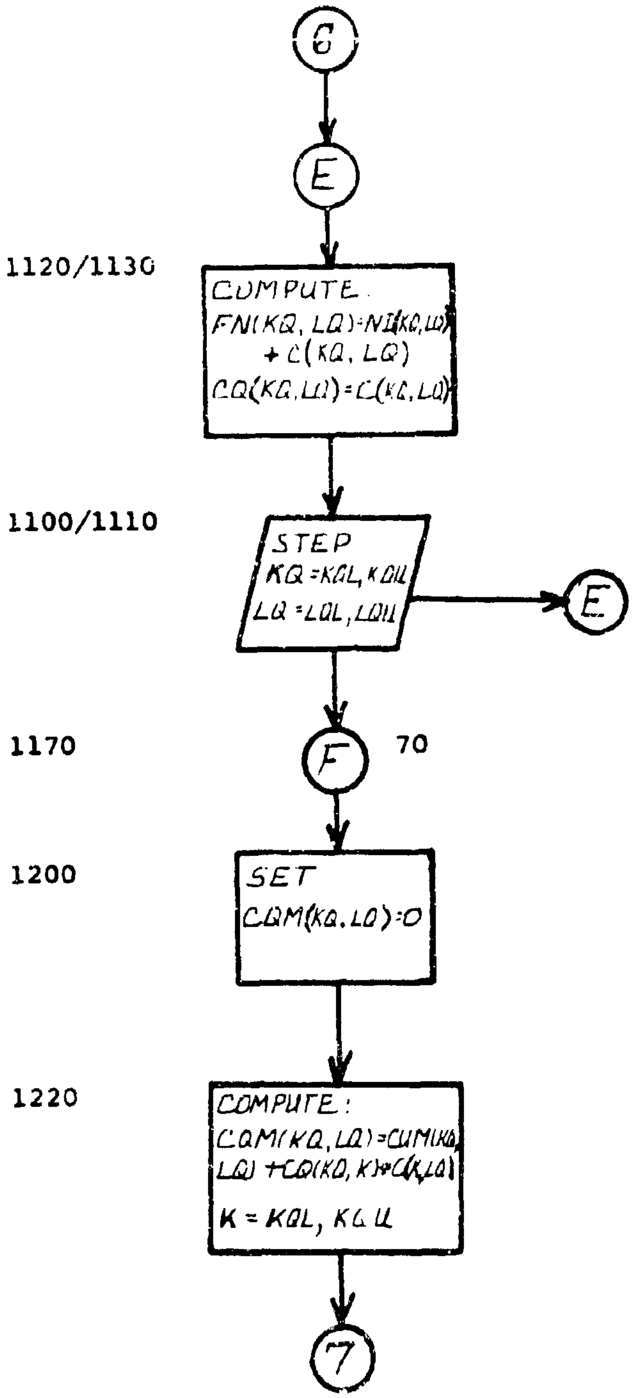
Print: Transition Matrix, C, in Canonical Form.

Print: Identity Matrix.

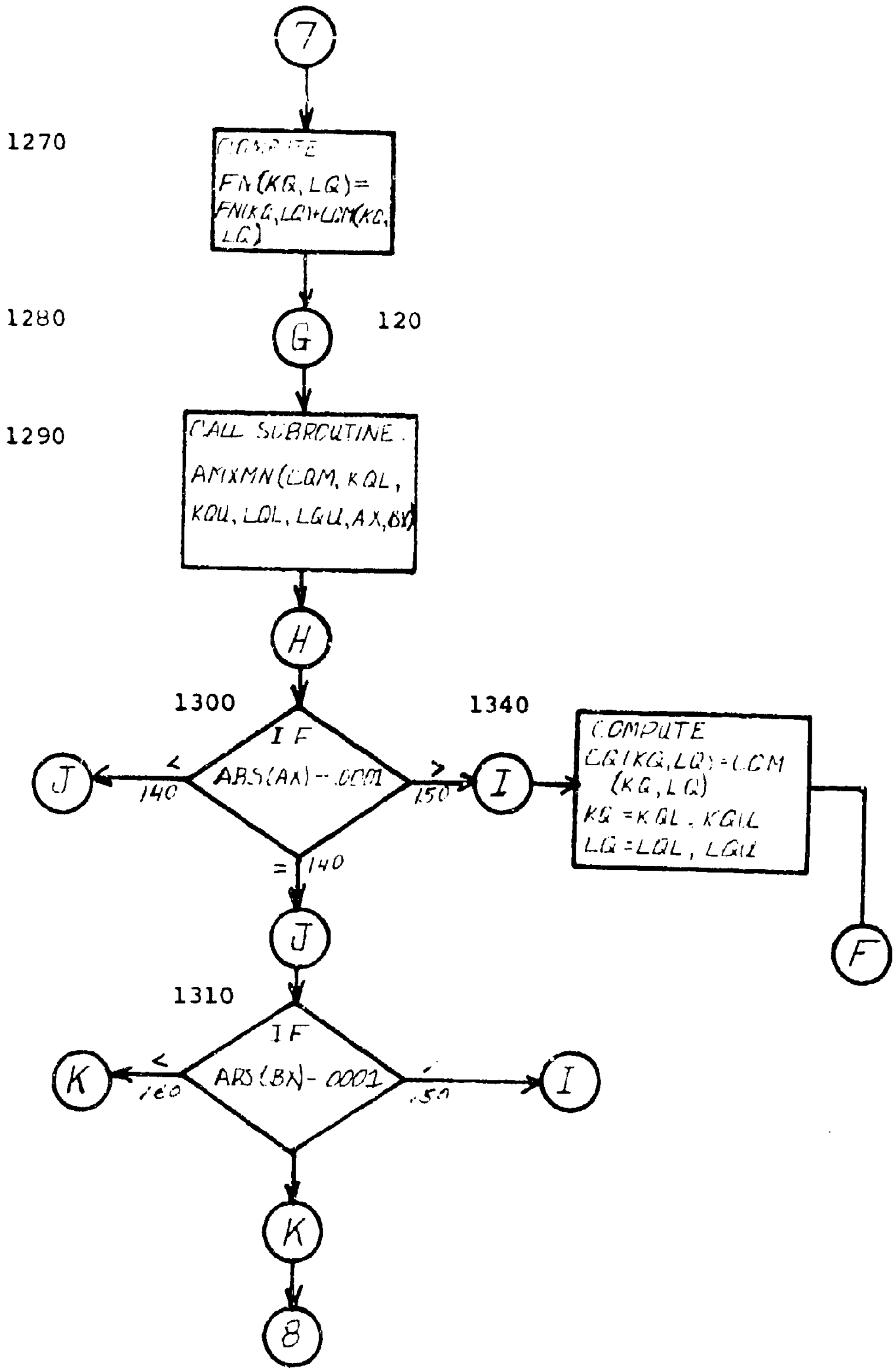
Print: Matrix of transient to absorbing probabilities.



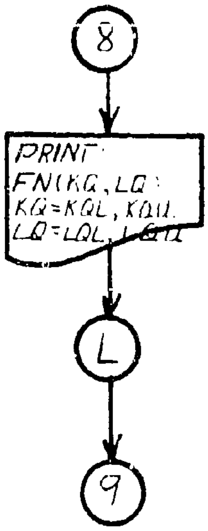
5
860/870
COMPUTE
KQN = KQU-KQL+1
LQN = LQU-LQL+1
900
PRINT
C(KQ, LQ)
KQ=KQL,KQU
LQ=LQL,LQU
Print: Matrix of transient to transient probabilities.
D
1010
SET
NI(KQ,LQ)=0
980
IF
(KQ-LQ)
<
20
>
20
=
10
1010
SET
NI(KQ,LQ)=0
990
SET
NI(KQ,LQ)=1
960/970
STEP
KQ=KQL,KQU
LQ=LQL,LQU
D
6



6
E
1120/1130
COMPUTE
FN(KQ, LQ)=NI(KQ,LQ)
+ C(KQ, LQ)
CQ(KQ,LQ)=C(KQ,LQ)
1100/1110
STEP
KQ=KQL, KQU
LQ=LQL, LQU
E
1170
F
70
1200
SET
CQM(KQ,LQ)=0
1220
COMPUTE:
CQM(KQ,LQ)=CQM(KQ,
LQ)+CQ(KQ,K)*C(K,LQ)
K=KQL, KQU
7


7
1270
COMPUTE
FN(KQ, LQ)=
FN(KQ,LQ)+LQM(KQ,
LQ)
1280
G
120
1290
CALL SUBROUTINE
AMXMN(LQM, KQL,
KQU, LQL, LQU, AX, BX)
H
1300
IF
ABS(AX)-.0001
<
140
J
>
150
I
1340
COMPUTE
LQ(KQ,LQ)=LQM
(KQ, LQ)
KQ=KQL, KQU
LQ=LQL, LQU
F
=
140
J
1310
IF
ABS(BX)-.0001
<
160
K
150
I
K
8

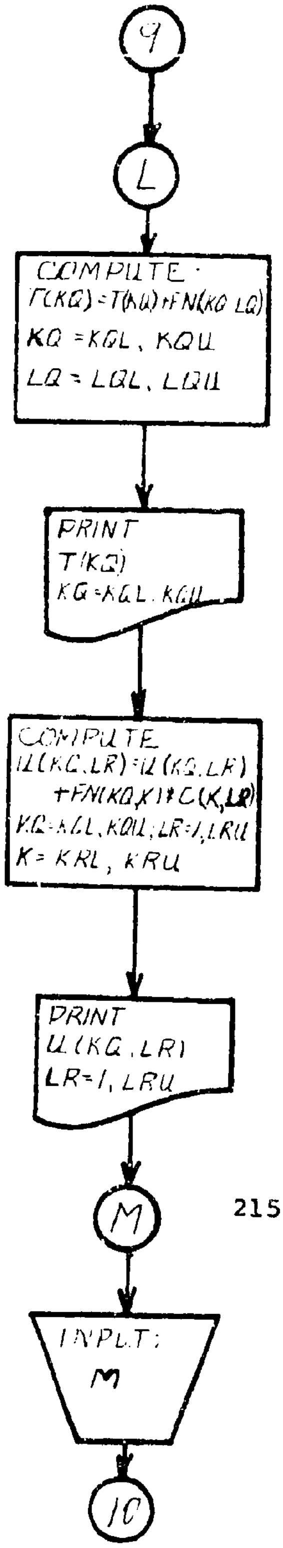
1370



1400

Print: Fundamental Matrix

MARK2



2120 Print: Matrix of Absorption times.

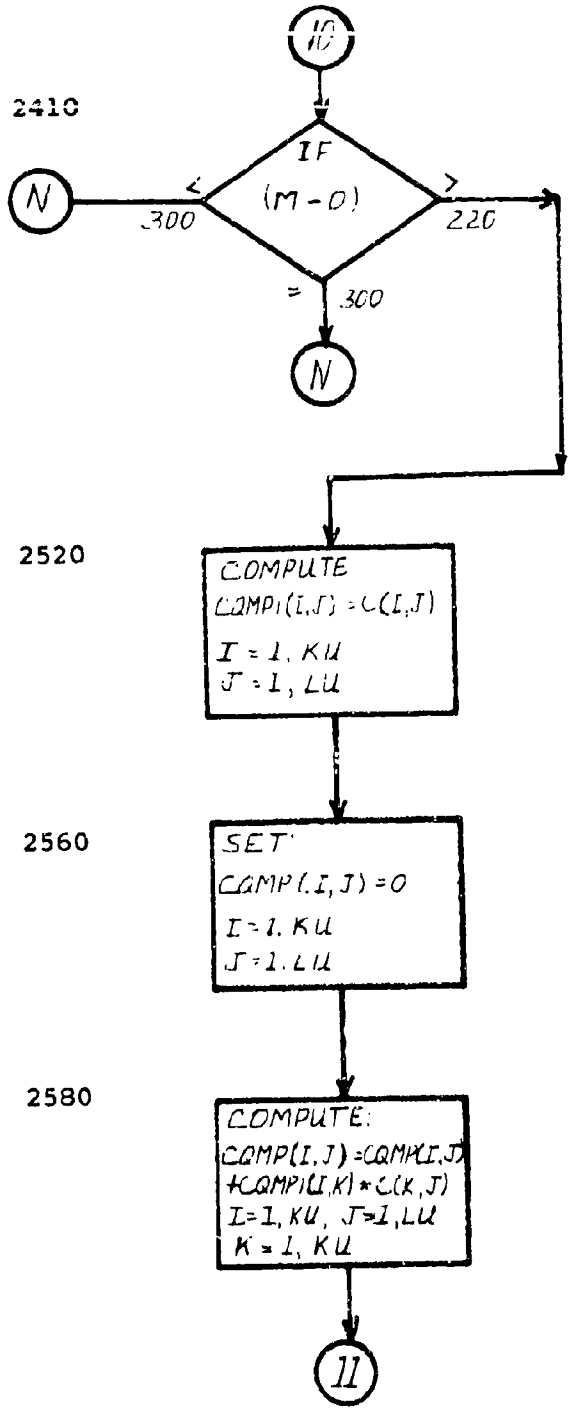
2140

2270

2310 Print: Matrix of Absorption probabilities.

2320 215

2400 Input: M, qty of steps.


10
2410
IF
(M - O)
N
<
300
>
220
=
300
N
2520
COMPUTE
COMP1(I,J) = C(I,J)
I = 1, KU
J = 1, LU
2560
SET
COMP(I,J) = 0
I = 1, KU
J = 1, LU
2580
COMPUTE:
COMP(I,J) = COMP(I,J)
+COMP1(I,K) * C(K,J)
I = 1, KU, J = 1, LU
K = 1, KU
11

2630

2670

Print: Transient probability Matrix for M steps.

2740

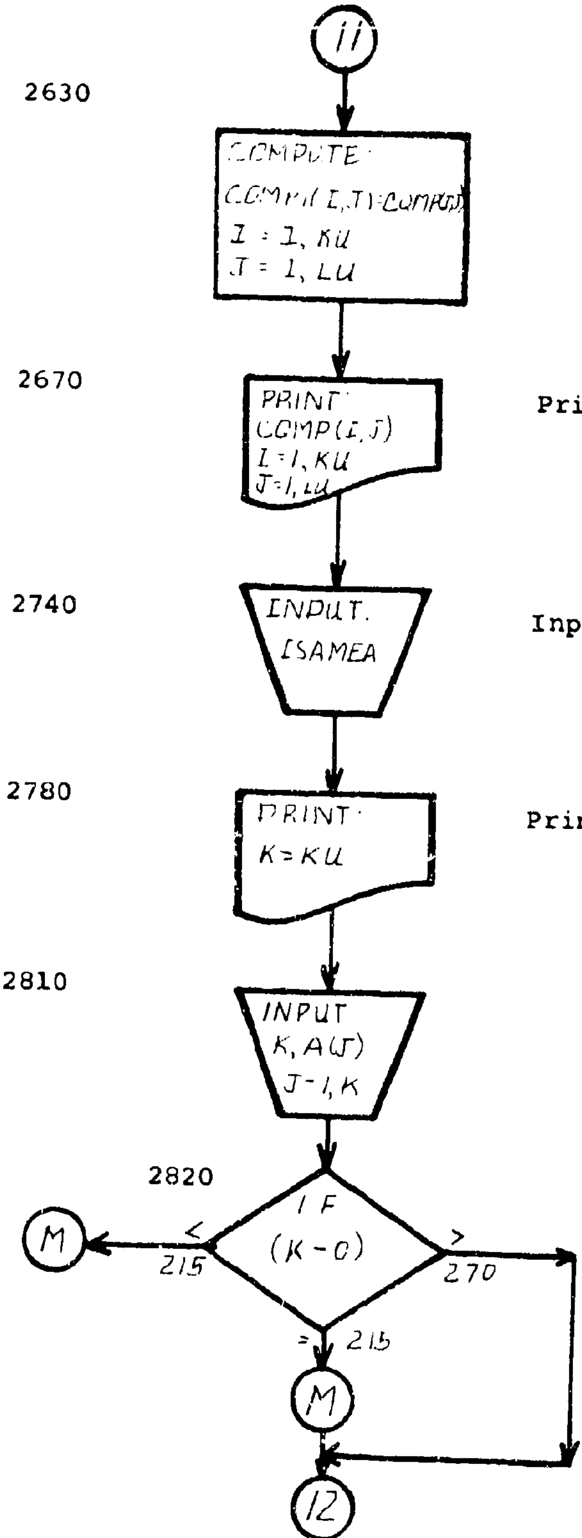
Input: 0 for same Vector A, 1 for new Vector A.

2780

Print: Qty of elements, K, in Vector A.

2810

2820

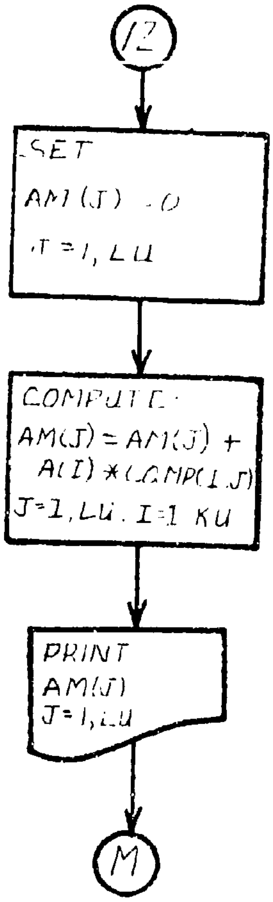


2910

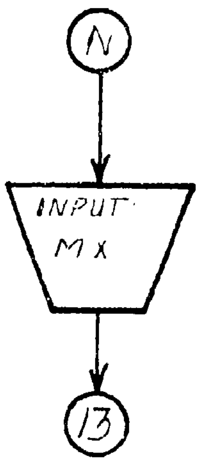
2940

2970

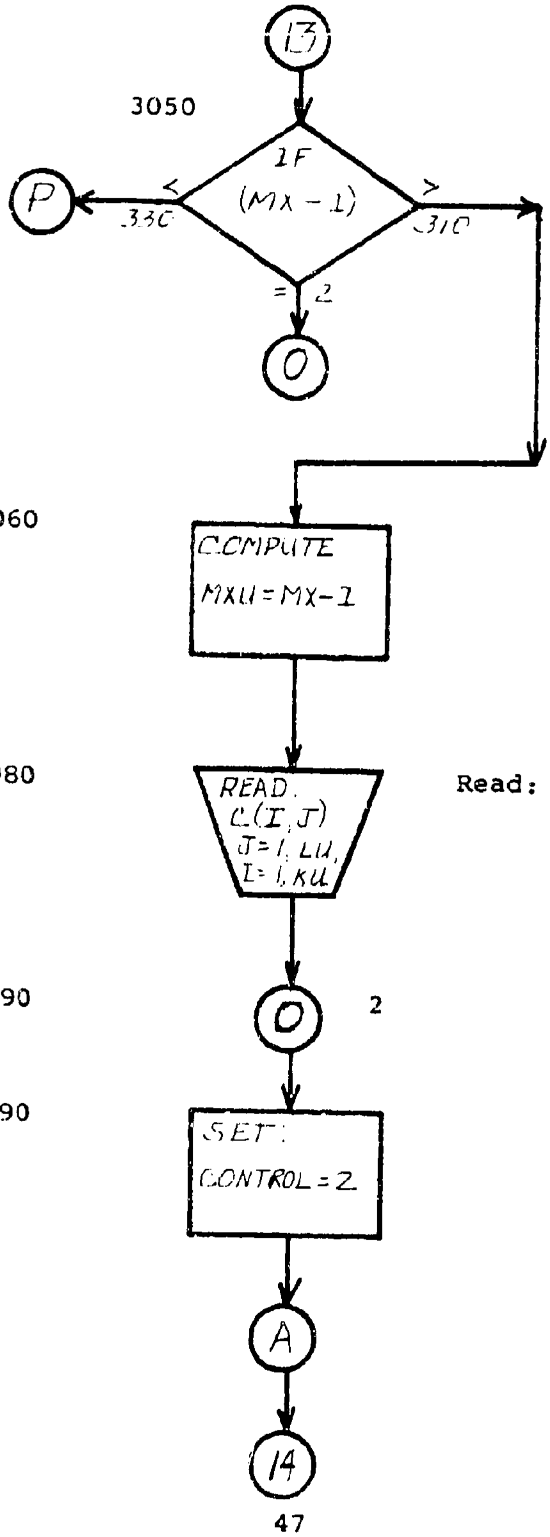
2990



Print: Vector, AM, for State Space M steps later.



Input: MX
0 to end Computation,
1 to Do next set data,
S+1 to skip S sets of data and do next set of data.



3050

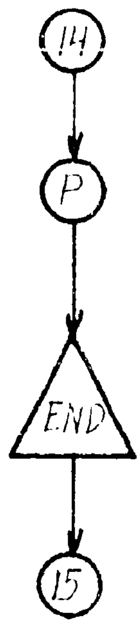
3060

3080

Read: Data Sets to be skipped.

3090 2

3090



3110

330

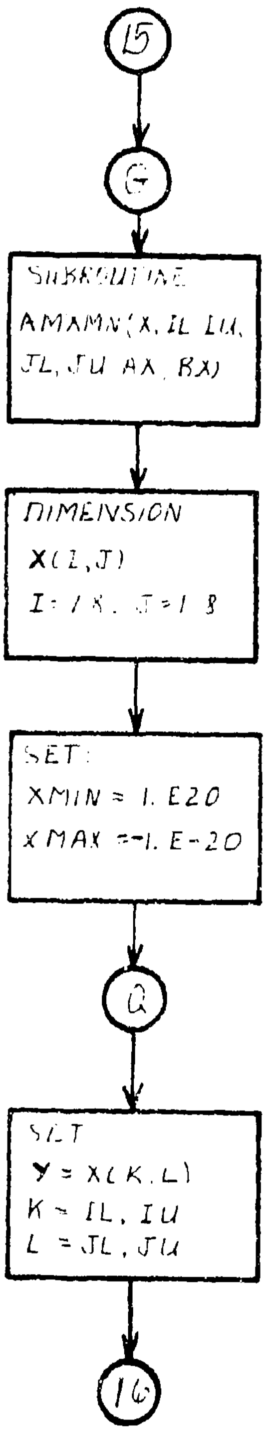
3110

4000

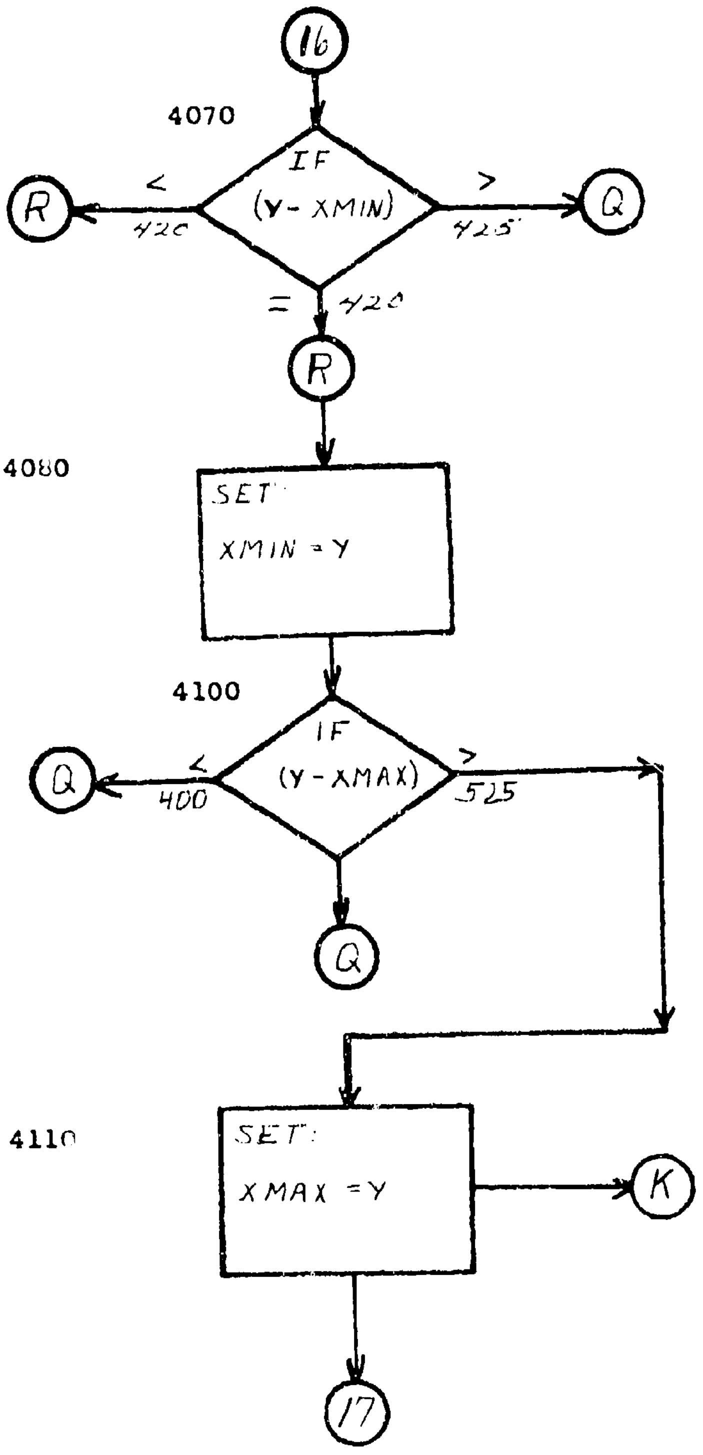
4010

4020/4030

4060

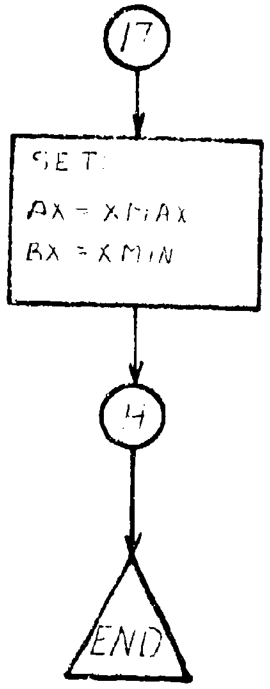


Obtains Max and Min Entry in Array X.


16
4070
IF
(Y - XMIN)
<
420
R
>
425
Q
=
420
R
4080
SET:
XMIN = Y
4100
IF
(Y - XMAX)
<
400
Q
>
525
Q
4110
SET:
XMAX = Y
K
17

4130/4140

4160



H. COMPUTER FORTRAN PROGRAM MARK 1/2/3

MARK1

```
100 PRINT "SIMULATIØN WITH"
110 PRINT "MARKØV TRANSITIØN MATRIX MØDEL ØF"
120 PRINT "A REQUISITIØN PRØCESSING SYSTEM"
130 PRINT "BY IRWIN GØØDMAN"
140 PRINT "INPUT, C, TRANSITIØN MATRIX IN CANØNICAL FØRM"
150 PRINT "IN LINES 1420 THRU 1800 BY RØWS"
160 PRINT "PRECEDE C MATRIX DATA WITH NAMES ØF STATES"
170 PRINT "RIGHT JUSTIFY BETWEEN CØMMAS WITH SPACES ADDED FØR"
180 PRINT "7 CHARACTER FØRMAT."
190 PRINT "EXAMPLE:   HØME,    BAR, CØRNR2,ETC. FØLLØWED BY DATA
220 CØNTRØL = 1
230 DIMENSIØN C(8,8),CQ",CQM",CQMP",CQMP1",NI",FN",U",X",T(8),A",AM"
240 DIMENSIØN STATE(8)
250 PRINT "INPUT QTY ØF ABSØRBING STATES=IABSØR"
260 INPUT, IABSØR
270 PRINT "INPUT QTY ØF TRANSIENT STATES=ITRANS"
280 INPUT, ITRANS
290C CØMPUTE QTY ØF RØWS AND CØLUMNS IN MATRIX C,
300 KU=IABSØR + ITRANS
310 LU=IABSØR + ITRANS
320 PRINT "QTY ØF RØWS IN MATRIX C =",KU
330 PRINT "QTY ØF CØLUMNS IN MATRIX C =",LU
340 1 DØ 65 I=1,8
350 T(I)=0; 65 AM(I)=0
360 DØ 66 I=1,8
370 DØ 66 J=1,8
380 CQM(I,J)=0; U(I,J)=0; 66 CQMP(I,J)=0
390 GØ TØ (3,4), CØNTRØL
400 3 READ, (STATE(J),J=1,LU)
410 4
420 READ, ((C(I,J),J=1,LU),I=1,KU)
430 PRINT
440 PRINT
450 PRINT "TRANSITIØN MATRIX, C, IN CANØNICAL FØRM"
460 PRINT
470 PRINT BB, "STATE  ",(STATE(J),J=1,LU)
480 BB:FØRMAT(10A7)
490 DØ B, I=1,KU
500 B: PRINT AA, STATE(I),(C(I,J),J=1,LU)
510 AA: FØRMAT(A7,10F7.3)
520 PRINT
530 PRINT
540 PRINT "PARTITIØN TRANSITIØN MATRIX (CANØNICAL FØRM)"
550 PRINT "INTØ FØLLØWING MATRICIES:"
560 PRINT "CI: IDENTITY MATRIX"
570 PRINT
580 KIU=IABSØR
590 LIU=IABSØR
600 PRINT BB, "STATE  ",(STATE(LI),LI=1,LIU)
610 DØ D, K"
```

H. COMPUTER FORTRAN PROGRAM MARK 1/2/3 (CONTD)

MARK1 CØNTINUED

```
620 D: PRINT AA,STATE(KI),(C(KI,LI),LI=1,LIU)
630 PRINT
640 PRINT
650 PRINT "CR: MATRIX ØF TRANSIENT TØ ABSØRBING PRØBABILITIES-"
660 PRINT "PRØBABILITY ØF GØING FRØM TRANSIENT STATE"
670 PRINT "TØ ABSØRBING STATE"
680 PRINT
690 KRL=IABSØR + 1
700 KRU=KU
710 LRU=IABSØR
720 KRN=KRU-KRL + 1
730 PRINT BB, "STATE   ", (STATE(LR),LR=1,LRU)
740 DØ E, KR=KRL,KRU
750 E: PRINT AA, STATE(KR), (C(KR,LR),LR=1,LRU)
760 PRINT
770 PRINT
780 PRINT "CQ: MATRIX ØF TRANSIENT TØ TRANSIENT PRØBABILITIES-"
790 PRINT "PRØBABILITY ØF GØING FRØM TRANSIENT STATE"
800 PRINT "TØ TRANSIENT STATE"
810 PRINT
820 KQL=IABSØR + 1
830 KQU=KU
840 LQL=IABSØR + 1
850 LQU=LU
860 KQN=KQU - KQL + 1
870 LQN=LQU - LQL + 1
880 PRINT BB, "STATE   ", (STATE(LQ), LQ=LQL,LQU)
890 DØ F, KQ=KQL,KQU
900 F: PRINT AA, STATE(KQ),(C(KQ,LQ),LQ=LQL,LQU)
910 PRINT
920 PRINT
930 PRINT "CØMPUTATIØN ØF THE FUNDAMENTAL MATRIX"
940C ESTABLISH IDENTITY MATRIX, NI, FØR CØMPUTATIØN ØF
950C FUNDAMENTAL MATRIX, FN.
960 DØ 40 KQ=KQL,KQU
970 DØ 30 LQ=LQL,LQU
980 IF (KQ-LQ) 20,10,20
990 10: NI(KQ,LQ)=1
1000 GØ TØ 30
1010 20: NI(KQ,LQ)=0
1020 30: CØNTINUE
1030 40: CØNTINUE
1040 PRINT "FN: FUNDAMENTAL MATRIX-"
1050 PRINT "EACH ELEMENT IS THE EXPECTED NUMBER ØF TIMES"
1060 PRINT "IN STATE J(CØLUMN) BEFØRE BEING ABSØRBED"
1070 PRINT "GIVEN THAT THE PRESENT STATE IS I(RØW)"
1080 PRINT
1090 PRINT BB, "STATE   ", (STATE(LQ),LQ=LQL,LQU)
1100 DØ 50 KQ=KQL,KQU
1110 DØ 50 LQ=LQL,LQU
```

H. COMPUTER FORTRAN PROGRAM MARK 1/2/3 (CONT'D.

MARK1 CØNTINUED

```
FN(KQ,LQ)=NI(KQ,LQ) + C(KQ,LQ)
CQ(KQ,LQ)=C(KQ,LQ)
50 CØNTINUE
C CØMPUTE THE FUNDAMENTAL MATRIX, FN, EQUAL TØ THE INVERSE
C ØF FNI=NI-CQ, BY SERIES APPRØXIMATIØN
70
DØ 90 KQ=KQL,KQU
DØ 90 LQ=LQL,LQU
CQM(KQ,LQ)=0
DØ 80 K=KQL,KQU
CQM(KQ,LQ)=CQM(KQ,LQ)+CQ(KQ,K)*C(K,LQ)
80 CØNTINUE
90 CØNTINUE
DØ 120 KQ=KQL,KQU
DØ 120 LQ=LQL,LQU
FN(KQ,LQ)=FN(KQ,LQ)+CQM(KQ,LQ)
120 CØNTINUE
CALL AMXMN(CQM,KQL,KQU,LQL,LQU,AX,BX)
IF (ABS(AX)-.0001) 140,140,150
140 IF(ABS(BX)-.0001) 160,160,150
150 DØ 500 KQ=KQL,KQU
DØ 500 LQ=LQL,LQU
500 CQ(KQ,LQ)=CQM(KQ,LQ)
GØ TØ 70
160 DØ G,KQ=KQL,KQU
G: PRINT AA, STATE(KQ), (FN(KQ,LQ),LQ=LQL,LQU)
PRINT
PRINT
$USE MARK2
$DATA
   HØME,   BAR, CØRNR2, CØRNR3, CØRNR4, CØRNR5, CØRNR6, CØRNR7
1.,7*0.
0.,1.,6*0.
.75,2*0.,.25,4*0.
2*0.,.75,0.,.25,3*0.
3*0.,.75,0.,.25,2*0.
4*0.,.75,0.,.25,0.
5*0.,.75,0.,.25,0.,.25,4*0.,.75,0.
 CØMPLT, FRSPAS, SUBPAS
1.,0.,0.
.99,0.,.01
.75,0.,.25
1.,0.,0.
.95,0.,.05
.75,0.,.25
1.,0.,0.
.90,0.,.10
.75,0.,.25
1.,0.,0.
.85,0.,.15
```

II. COMPUTER FORTRAN PROGRAM MARK 1/2/3 (CONTD)

MARK1 CONTINUED

```
1472 .75,0.,.25
1480 1.,0.,0.
1481 .80,0.,.20
1482 .75,0.,.25
1490 1.,0.,0.
1491 .75,0.,.25
1492 .75,0.,.25
1500 1.,0.,0.
1501 .50,0.,.50
1502 .75,0.,.25
1510 1.,0.,0.
1511 .25,0.,.75
1512 .75,0.,.25
1520 1.,0.,0.
1521 .01,0.,.99
1522 .75,0.,.25
1530 1.,0.,0.
1531 .001,0.,.999
1532 .75,0.,.25
1801      MRØ, PASØRD, REJCUS, BACKØR, FRSPAS, SUBPAS
1802 1.,5*0.,0.,1.,4*0.,2*0.,1.,3*0.,3*0.,1.,2*0.
1803 .55,.02,.03,.22,0.,.18,.06,.09,.22,.40,0.,.23
```

MARK2

```
2000 PRINT "CØMPUTATIØN ØF THE MATRIX ØF ABSØRPTIØN TIMES"
2010 PRINT "T: MATRIX ØF ABSØRPTIØN TIMES-"
2020 PRINT "EACH ELEMENT IS THE MEAN TIME TØ ABSØRPTIØN"
2030 PRINT "(NUMBER ØF STATES PASSED THRØUGH, INCLUDING FINAL STATE"
2040  PRINT "AND NØT INCLUDING INITIAL STATE, INØRDER TØ BE ABSØRBED)"
 2050 PRINT
2060C CØMPUTE T=FN TIMES C WHERE C IS A CØLUMN VECTØR ØF 1'S.
2070C THE ELEMENTS IN MATRIX T ARE EQUAL TØ THE RØW SUM FØR EACH
2080C RØW ØF FN.
2090 PRINT BB, "STATE  "
2100 DØ 180 KQ=KQL,KQU
2110 DØ 170 LQ=LQL,LQU
2120 T(KQ)=T(KQ) + FN(KQ,LQ)
2130 170 CØNTINUE
2140 PRINT AA, STATE(KQ),T(KQ)
2150  180 CØNTINUE
2160 PRINT
2170 PRINT
```

H. COMPUTER FORTRAN PROGRAM MARK 1/2/3 (CONTD)

MARK2 CØNTINUED

```
2180 PRINT "CØMPUTATIØN ØF THE MATRIX ØF ABSØRPTIØN PRØBABILITIES"
2190 PRINT "U. MATRIX ØF ABSØRPTIØN PRØBABILITIES"
2200 PRINT "PRØBABILITY ØF BEING ABSØRBED,"
2210 PRINT "GIVEN IT WAS INITIALLY IN A TRANSIENT STATE"
2220 PRINT
2230C MULTIPLY MATRIX FN TIMES CR.
2240 DØ 190 KQ=KQL,KQU
2250 DØ 190 LR=1,LRU
2260 DØ 190 K=KRL,KRU
2270 U(KQ,LR)=U(KQ,LR) + FN(KQ,K)*C(K,LR)
2280 190 CØNTINUE
2290 PRINT BB, "STATE  ", (STATE(LR),LR=1,LRU)
2300 DØ GG, KQ=KQL,KQU
2310 GG: PRINT AA, STATE(KQ),(U(KQ,LR),LR=1,LRU)
2320 215
2330 PRINT
2340 PRINT
2350 PRINT "CØMPUTATIØN ØF TRANSIENT PRØBABILITY MATRIX FØR M STEPS"
2360 PRINT "INPUT NUMBER ØF STEPS, M,"
2370 PRINT "THAT MATRIX ØF TRANSITIØN PRØBABILITIES"
2380 PRINT "SHØULD BE CØMPUTED FØR"
2390 PRINT " INPUT M=0 IF CØMPUTATIØN NØT DESIRED"
2400 INPUT, M
2410 IF(M-0) 300,300,220
2420C GØ TØ END FØR M=0
2430 220 PRINT
2440 PRINT
2450 PRINT "CMP: MATRIX ØF TRANSITIØN"
2460 PRINT "PRØBABILITIES FØR M STEPS,M=",M
2470 PRINT "PRØBABILITY ØF GØING FRØM STATE TØ STATE"
2480 PRINT
2490C CØMPUTE C TØ THE M PØWER.
2500 DØ 225 I=1,KU
2510 DØ 225 J=1,LU
2520 225 CQMP1(I,J)=C(I,J)
2530 DØ 260 L=2,M
2540 DØ 250 I=1,KU
2550 DØ 250 J=1,LU
2560 CQMP(I,J)=0
2570 DØ 230 K=1,KU
2580 CQMP(I,J)=CQMP(I,J)+CQMP1(I,K)*C(K,J)
2590 230 CØNTINUE
2600 250 CØNTINUE
2610 DØ 255 I=1,KU
2620 DØ 255 J=1,LU
2630 255 CQMP1(I,J)=CQMP(I,J)
2640 260 CØNTINUE
2650 PRINT BB, "STATE  ", (STATE(J),J=1,LU)
2660 DØ H,I=1,KU
2670 H: PRINT AA, STATE(I), (CQMP(I,J),J=1,LU)
```

MARK2 CØNTINUED

```
PRINT
PRINT
PRINT "CØMPUTATIØN ØF STATE SPACE, M STEPS LATER,"
PRINT "GIVEN INITIAL STATE"
PRINT "INPUT ISAMEA=0 IF SAME RØW VECTØR TØ BE USED"
PRINT "INPUT ISAMEA=1 FØR NEW A(K) VECTØR INPUT."
INPUT,ISAMEA
IF(ISAMEA-1) 270,265,270
265
PRINT "INPUT INITIAL STATE AS A RØW VECTØR IN FØLLØWING FØRM"
PRINT "K=",KU
PRINT "INPUT K=0 IF CØMPUTATIØN NØT DESIRED"
PRINT "INPUT K,A(1),A(2),A(3),...,A(K)"
INPUT,K,(A(J),J=1,K)
IF(K-0) 215,215,270
C REPEAT M STEP CØMPUTATIØN.
270 PRINT
PRINT
PRINT "AM: RØW VECTØR FØR STATE SPACE M STEPS LATER,M=",M
PRINT "GIVEN INITIAL STATE A"
C CØMPUTE AM=A TIMES CQMP.
PRINT
DØ 275 J=1,LU
275 AM(J)=0
DØ 290 J=1,LU
DØ 290 I=1,KU
AM(J)=AM(J)+A(I)*CQMP(I,J)
290 CØNTINUE
PRINT BB, "STATE   ", (STATE(J),J=1,LU)
PRINT CC, (AM(J),J=1,LU)
CC: FØRMAT(7X,10F7.3)
GØ TØ 215
300 PRINT "INPUT MX=0 TØ END CØMPUTATIØN"
PRINT "MX=1 TØ DØ NEXT SET ØF DATA"
PRINT "MX=S+1 TØ SKIP S SETS ØF DATA"
PRINT "AND DØ NEXT SET ØF DATA"
INPUT,MX
IF(MX-1) 330,2,310
310 MXU=MX-1
DØ 320 K=1,MXU
320 READ,((C(I,J),J=1,LU),I=1,KU)
2 CØNTRØL = 2
GØ TØ 1
330 END
$USE MARK3
```

H. COMPUTER FORTRAN PROGRAM MARK 1/2/3 (CONTD)

MARK3

```
SUBRØUTINE AMXMN(X,IL,IU,JL,JU,AX,BX)
DIMENSIØN X(8,8)
XMIN=1.E20
XMAX=-1.E-20
DØ 400 K=IL,IU
DØ 400 L=JL,JU
Y=X(K,L)
IF(Y-XMIN) 420,420,425
420 XMIN=Y
425 CØNTINUE
IF(Y-XMAX) 400,400,525
525 XMAX=Y
400 CØNTINUE
AX=XMAX
BX=XMIN
RETURN
END
```

I. COMPUTER TIME SHARING TERMINAL PRINT-OUTS

I.1 Classical Random Walk

```
MARK1      13:11    MØN.---07/08/68

IN MARK2
IN .FIRST
IN MARK3
IN .FIRST

SIMULATIØN WITH
MARKØV TRANSITIØN MATRIX MØDEL ØF
A REQUISITIØN PRØCESSING SYSTEM
BY IRWIN GØØDMAN
INPUT, C, TRANSITIØN MATRIX IN CANØNICAL FØRM
IN LINES 1420 THRU 1800 BY RØWS
PRECEDE C MATRIX DATA WITH NAMES ØF STATES
RIGHT JUSTIFY BETWEEN CØMMAS WITH SPACES ADDED FØR
7 CHARACTER FØRMAT.
EXAMPLE:   HØME,    BAR, CØRNR2,ETC. FØLLØWED BY DATA
INPUT QTY ØF ABSØRBING STATES=IABSØR
? 72
INPUT QTY ØF TRANSIENT STATES=ITRANS
? 76
QTY ØF RØWS IN MATRIX C =         8
QTY ØF CØLUMNS IN MATRIX C =        8


TRANSITIØN MATRIX, C, IN CANØNICAL FØRM
```

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
HØME	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BAR	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
CØRNR2	.750	0.000	0.000	.250	0.000	0.000	0.000	0.000
CØRNR3	0.000	0.000	.750	0.000	.250	0.000	0.000	0.000
CØRNR4	0.000	0.000	0.000	.750	0.000	.250	0.000	0.000
CØRNR5	0.000	0.000	0.000	0.000	.750	0.000	.250	0.000
CØRNR6	0.000	0.000	0.000	0.000	0.000	.750	0.000	.250
CØRNR7	0.000	.250	0.000	0.000	0.000	0.000	.750	0.000

```
PARTITIØN TRANSITIØN MATRIX (CANØNICAL FØRM)
INTØ FØLLØWING MATRICIES:
CI: IDENTITY MATRIX
```

STATE	HØME	BAR
HØME	1.000	0.000
BAR	0.000	1.000

I. COMPUTER TIME SHARING TERMINAL PRINT-OUTS (CONTD)

I.1 Classical Random Walk

CR: MATRIX ØF TRANSIENT TØ ABSØRBING PRØBABILITIES-
PRØBABILITY ØF GØING FRØM TRANSIENT STATE
TØ ABSØRBING STATE

STATE	HØME	BAR
CØRNR2	.750	0.000
CØRNR3	0.000	0.000
CØRNR4	0.000	0.000
CØRNR5	0.000	0.000
CØRNR6	0.000	0.000
CØRNR7	0.000	.250

CQ: MATRIX ØF TRANSIENT TØ TRANSIENT PRØBABILITIES-
PRØBABILITY ØF GØING FRØM TRANSIENT STATE
TØ TRANSIENT STATE

STATE	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
CØRNR2	0.000	.250	0.000	0.000	0.000	0.000
CØRNR3	.750	0.000	.250	0.000	0.000	0.000
CØRNR4	0.000	.750	0.000	.250	0.000	0.000
CØRNR5	0.000	0.000	.750	0.000	.250	0.000
CØRNR6	0.000	0.000	0.000	.750	0.000	.250
CØRNR7	0.000	0.000	0.000	0.000	.750	0.000

CØMPUTATIØN ØF THE FUNDAMENTAL MATRIX
FN: FUNDAMENTAL MATRIX-
EACH ELEMENT IS THE EXPECTED NUMBER ØF TIMES
IN STATE J(CØLUMN) BEFØRE BEING ABSØRBED
GIVEN THAT THE PRESENT STATE IS I(RØW)

STATE	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
CØRNR2	1.332	.443	.146	.048	.015	.004
CØRNR3	1.328	1.771	.586	.190	.059	.015
CØRNR4	1.317	1.757	1.903	.618	.190	.048
CØRNR5	1.284	1.713	1.855	1.903	.586	.146
CØRNR6	1.186	1.581	1.713	1.757	1.771	.443
CØRNR7	.889	1.186	1.284	1.317	1.328	1.332

CØMPUTATIØN ØF THE MATRIX ØF ABSØRPTIØN TIMES
T: MATRIX ØF ABSØRPTIØN TIMES-
EACH ELEMENT IS THE MEAN TIME TØ ABSØRPTIØN
(NUMBER ØF STATES PASSED THRØUGH, INCLUDING FINAL STATE
AND NØT INCLUDING INITIAL STATE, INØRDER TØ BE ABSØRBED)

STATE	
CØRNR2	1.987
CØRNR3	3.949
CØRNR4	5.833
CØRNR5	7.487
CØRNR6	8.450
CØRNR7	7.337

I.1 Classical Random Walk

```
COMPUTATIØN ØF THE MATRIX ØF ABSØRPTIØN PRØBABILITIES
U, MATRIX ØF ABSØRPTIØN PRØBABILITIES
PRØBABILITY ØF BEING ABSØRBED,
GIVEN IT WAS INITIALLY IN A TRANSIENT STATE

STATE    HØME   BAR
 CØRNR2   .999   .001
 CØRNR3   .996   .004
 CØRNR4   .988   .012
 CØRNR5   .963   .037
 CØRNR6   .889   .111
 CØRNR7   .667   .333


CØMPUTATIØN ØF TRANSIENT PRØBABILITY MATRIX FØR M STEPS
INPUT NUMBER ØF STEPS, M,
THAT MATRIX ØF TRANSITIØN PRØBABILITIES
SHØULD BE CØMPUTED FØR
 INPUT M=0 IF CØMPUTATIØN NØT DESIRED
? ?2


CMP: MATRIX ØF TRANSITIØN
PRØBABILITIES FØR M STEPS,M=        2
PRØBABILITY ØF GØING FRØM STATE TØ STATE

STATE    HØME    BAR     CØRNR2 CØRNR3 CØRNR4 CØRNR5 CØRNR6 CØRNR7
 HØME    1.000   0.000   0.000  0.000  0.000  0.000  0.000  0.000
 BAR     0.000   1.000   0.000  0.000  0.000  0.000  0.000  0.000
 CØRNR2   .750   0.000    .187  0.000   .062  0.000  0.000  0.000
 CØRNR3   .562   0.000   0.000   .375  0.000   .062  0.000  0.000
 CØRNR4  0.000   0.000    .562  0.000   .375  0.000   .062  0.000
 CØRNR5  0.000   0.000   0.000   .562  0.000   .375  0.000   .062
 CØRNR6  0.000    .062   0.000  0.000   .562  0.000   .375  0.000
 CØRNR7  0.000    .250   0.000  0.000  0.000   .562  0.000   .187


CØMPUTATIØN ØF STATE SPACE, M STEPS LATER,
GIVEN INITIAL STATE
INPUT ISAMEA=0 IF SAME RØW VECTØR TØ BE USED
INPUT ISAMEA=1 FØR NEW A(K) VECTØR INPUT.
1
? ?1
INPUT INITIAL STATE AS A RØW VECTØR IN FØLLØWING FØRM
K=      8
INPUT K=0 IF CØMPUTATIØN NØT DESIRED
INPUT K,A(1),A(2),A(3),...,A(K)
? ?8,0,0,.167,.167,.167,.167,.167,.167


AM, RØW VECTØR FØR STATE SPACE M STEPS LATER,M=        2
GIVEN INITIAL STATE A

STATE    HØME    BAR     CØRNR2 CØRNR3 CØRNR4 CØRNR5 CØRNR6 CØRNR7
          .219    .052    .125   .157   .167   .167   .073   .042
```

```
CØMPUTATIØN ØF TRANSIENT PRØBABILITY MATRIX FØR M STEPS
INPUT NUMBER ØF STEPS, M,
THAT MATRIX ØF TRANSITIØN PRØBABILITIES
SHØULD BE CØMPUTED FØR
 INPUT M=0 IF CØMPUTATIØN NØT DESIRED
? ?25


CMP: MATRIX ØF TRANSITIØN
PRØBABILITIES FØR M STEPS,M=       25
PRØBABILITY ØF GØING FRØM STATE TØ STATE
```

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
HØME	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BAR	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
CØRNR2	.999	.001	0.000	.000	0.000	.000	0.000	.000
CØRNR3	.995	.004	.001	0.000	.001	0.000	.000	0.000
CØRNR4	.986	.012	0.000	.002	0.000	.001	0.000	.000
CØRNR5	.959	.037	.003	0.000	.002	0.000	.001	0.000
CØRNR6	.884	.111	0.000	.004	0.000	.002	0.000	.000
CØRNR7	.660	.333	.003	0.000	.003	0.000	.001	0.000

```
CØMPUTATIØN ØF STATE SPACE, M STEPS LATER,
GIVEN INITIAL STATE
INPUT ISAMEA=0 IF SAME RØW VECTØR TØ BE USED
INPUT ISAMEA=1 FØR NEW A(K) VECTØR INPUT.
? ?0


AM, RØW VECTØR FØR STATE SPACE M STEPS LATER,M=       25
GIVEN INITIAL STATE A
```

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
	.916	.083	.001	.001	.001	.000	.000	.000

```
CØMPUTATIØN ØF TRANSIENT PRØBABILITY MATRIX FØR M STEPS
INPUT NUMBER ØF STEPS, M,
THAT MATRIX ØF TRANSITIØN PRØBABILITIES
SHØULD BE CØMPUTED FØR
 INPUT M=0 IF CØMPUTATIØN NØT DESIRED
? ?3


CMP: MATRIX ØF TRANSITIØN
PRØBABILITIES FØR M STEPS,M=        3
PRØBABILITY ØF GØING FRØM STATE TØ STATE
```

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
HØME	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BAR	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
CØRNR2	.891	0.000	0.000	.094	0.000	.016	0.000	0.000
CØRNR3	.562	0.000	.281	0.000	.141	0.000	.016	0.000
CØRNR4	.422	0.000	0.000	.422	0.000	.141	0.000	.016
CØRNR5	0.000	.016	.422	0.000	.422	0.000	.141	0.000
CØRNR6	0.000	.062	0.000	.422	0.000	.422	0.000	.094
CØRNR7	0.000	.297	0.000	0.000	.422	0.000	.281	0.000

```
CØMPUTATIØN ØF STATE SPACE, M STEPS LATER,
GIVEN INITIAL STATE
INPUT ISAMEA=0 IF SAME RØW VECTØR TØ BE USED
INPUT ISAMEA=1 FØR NEW A(K) VECTØR INPUT.
? ?0


AM. RØW VECTØR FØR STATE SPACE M STEPS LATER,M=       3
GIVEN INITIAL STATE A

STATE    HØME    BAR     CØRNR2 CØRNR3 CØRNR4 CØRNR5 CØRNR6 CØRNR7
          .313    .063    .117   .157   .164   .097   .073   .018


CØMPUTATIØN ØF TRANSIENT PRØBABILITY MATRIX FØR M STEPS
INPUT NUMBER ØF STEPS, M,
THAT MATRIX ØF TRANSITIØN PRØBABILITIES
SHØULD BE CØMPUTED FØR
 INPUT M=0 IF CØMPUTATIØN NØT DESIRED
? ?5


CMP: MATRIX ØF TRANSITIØN
PRØBABILITIES FØR M STEPS,M=       5
PRØBABILITY ØF GØING FRØM STATE TØ STATE

STATE    HØME    BAR     CØRNR2 CØRNR3 CØRNR4 CØRNR5 CØRNR6 CØRNR7
 HØME    1.000   0.000   0.000  0.000  0.000  0.000  0.000  0.000
 BAR     0.000   1.000   0.000  0.000  0.000  0.000  0.000  0.000
 CØRNR2   .943   0.000   0.000   .044  0.000   .012  0.000   .001
 CØRNR3   .773    .001    .132  0.000   .079  0.000   .015  0.000
 CØRNR4   .659    .004   0.000   .237  0.000   .088  0.000   .012
 CØRNR5   .316    .024    .316  0.000   .264  0.000   .079  0.000
 CØRNR6   .237    .086   0.000   .396  0.000   .237  0.000   .044
 CØRNR7  0.000    .314    .237  0.000   .316  0.000   .132  0.000

CØMPUTATIØN ØF STATE SPACE, M STEPS LATER,
GIVEN INITIAL STATE
INPUT ISAMEA=0 IF SAME RØW VECTØR TØ BE USED
INPUT ISAMEA=1 FØR NEW A(K) VECTØR INPUT.
? ?0


AM. RØW VECTØR FØR STATE SPACE M STEPS LATER,M=       5
GIVEN INITIAL STATE A

STATE    HØME    BAR     CØRNR2 CØRNR3 CØRNR4 CØRNR5 CØRNR6 CØRNR7
          .489    .072    .114   .113   .110   .056   .038   .009


CØMPUTATIØN ØF TRANSIENT PRØBABILITY MATRIX FØR M STEPS
INPUT NUMBER ØF STEPS, M,
THAT MATRIX ØF TRANSITIØN PRØBABILITIES
SHØULD BE CØMPUTED FØR
 INPUT M=0 IF CØMPUTATIØN NØT DESIRED
? ?0
```

1.2 Model A of Requisition Processing System

MARKI 13:45 MØN.---07/08/68

IN [illegible]
IN [illegible]
IN [illegible]
IN [illegible]

SIMULATIØN WITH
MARKØV TRANSITIØN MATRIX MØDEL ØF
A REQUISITIØN PRØCESSING SYSTEM
BY IRWIN GØØDMAN
INPUT, C, TRANSITIØN MATRIX IN CANØNICAL FØRM
IN LINES 1420 THRU 1800 BY RØWS
PRECEDE C MATRIX DATA WITH NAMES ØF STATES
RIGHT JUSTIFY BETWEEN CØMMAS WITH SPACES ADDED FØR
7 CHARACTER FØRMAT.
EXAMPLE: HØME, BAR, CØRNR2,ETC. FØLLØWED BY DATA
INPUT QTY ØF ABSØRBING STATES=IABSØR
? ?4
INPUT QTY ØF TRANSIENT STATES=ITRANS
? ?2
QTY ØF RØWS IN MATRIX C = 6
QTY ØF CØLUMNS IN MATRIX C = 6

TRANSITIØN MATRIX, C, IN CANØNICAL FØRM

STATE	MRØ	PASØRD	REJCUS	BACKØR	FRSPAS	SUBPAS
MRØ	1.000	0.000	0.000	0.000	0.000	0.000
PASØRD	0.000	1.000	0.000	0.000	0.000	0.000
REJCUS	0.000	0.000	1.000	0.000	0.000	0.000
BACKØR	0.000	0.000	0.000	1.000	0.000	0.000
FRSPAS	.550	.020	.030	.220	0.000	.180
SUBPAS	.060	.090	.220	.400	0.000	.230

PARTITIØN TRANSITIØN MATRIX (CANØNICAL FØRM)
INTØ FØLLØWING MATRICIES:
CI: IDENTITY MATRIX

STATE	MRØ	PASØRD	REJCUS	BACKØR
MRØ	1.000	0.000	0.000	0.000
PASØRD	0.000	1.000	0.000	0.000
REJCUS	0.000	0.000	1.000	0.000
BACKØR	0.000	0.000	0.000	1.000

```
CR: MATRIX ØF TRANSIENT TØ ABSØRBING PRØBABILITIES-
PRØBABILITY ØF GØING FRØM TRANSIENT STATE
TØ ABSØRBING STATE

STATE    MRØ    PASØRD REJCUS BACKØR
 FRSPAS   .550   .020   .030   .220
 SUBPAS   .060   .090   .220   .400


CQ: MATRIX ØF TRANSIENT TØ TRANSIENT PRØBABILITIES-
PRØBABILITY ØF GØING FRØM TRANSIENT STATE
TØ TRANSIENT STATE

STATE    FRSPAS SUBPAS
 FRSPAS  0.000   .180
 SUBPAS  0.000   .230


CØMPUTATIØN ØF THE FUNDAMENTAL MATRIX
FN. FUNDAMENTAL MATRIX-
EACH ELEMENT IS THE EXPECTED NUMBER ØF TIMES
IN STATE J(CØLUMN) BEFØRE BEING ABSØRBED
GIVEN THAT THE PRESENT STATE IS I(RØW)

STATE    FRSPAS SUBPAS
 FRSPAS  1.000   .234
 SUBPAS  0.000  1.299


CØMPUTATIØN ØF THE MATRIX ØF ABSØRPTIØN TIMES
T: MATRIX ØF ABSØRPTIØN TIMES-
EACH ELEMENT IS THE MEAN TIME TØ ABSØRPTIØN
(NUMBER ØF STATES PASSED THRØUGH, INCLUDING FINAL STATE
AND NØT INCLUDING INITIAL STATE, INØRDER TØ BE ABSØRBED)

STATE
 FRSPAS  1.234
 SUBPAS  1.299


CØMPUTATIØN ØF THE MATRIX ØF ABSØRPTIØN PRØBABILITIES
U. MATRIX ØF ABSØRPTIØN PRØBABILITIES
PRØBABILITY ØF BEING ABSØRBED,
GIVEN IT WAS INITIALLY IN A TRANSIENT STATE

STATE    MRØ    PASØRD REJCUS BACKØR
 FRSPAS   .564   .041   .081   .314
 SUBPAS   .078   .117   .286   .519


CØMPUTATIØN ØF TRANSIENT PRØBABILITY MATRIX FØR M STEPS
INPUT NUMBER ØF STEPS, M,
THAT MATRIX ØF TRANSITIØN PRØBABILITIES
SHØULD BE CØMPUTED FØR
 INPUT M=0 IF CØMPUTATIØN NØT DESIRED
? ?2
```

```
CMP: MATRIX ØF TRANSITIØN
PRØBABILITIES FØR M STEPS,M=        2
PRØBABILITY ØF GØING FRØM STATE TØ STATE

STATE    MRØ    PASØRD REJCUS BACKØR FRSPAS SUBPAS
 MRØ     1.000  0.000  0.000  0.000  0.000  0.000
 PASØRD  0.000  1.000  0.000  0.000  0.000  0.000
 REJCUS  0.000  0.000  1.000  0.000  0.000  0.000
 BACKØR  0.000  0.000  0.000  1.000  0.000  0.000
 FRSPAS   .561   .036   .070   .292  0.000   .041
 SUBPAS   .074   .111   .271   .492  0.000   .053

CØMPUTATIØN ØF STATE SPACE, M STEPS LATER,
GIVEN INITIAL STATE
INPUT ISAMEA=0 IF SAME RØW VECTØR TØ BE USED
INPUT ISAMEA=1 FØR NEW A(K) VECTØR INPUT.
? ?1
INPUT INITIAL STATE AS A RØW VECTØR IN FØLLØWING FØRM
K=       6
INPUT K=0 IF CØMPUTATIØN NØT DESIRED
INPUT K,A(1),A(2),A(3),...,A(K)
? ?6,0,0,0,0,.80,.20

AM, RØW VECTØR FØR STATE SPACE M STEPS LATER,M=        2
GIVEN INITIAL STATE A

STATE    MRØ    PASØRD REJCUS BACKØR FRSPAS SUBPAS
          .463   .051   .110   .332  0.000   .044

CØMPUTATIØN ØF TRANSIENT PRØBABILITY MATRIX FØR M STEPS
INPUT NUMBER ØF STEPS, M,
THAT MATRIX ØF TRANSITIØN PRØBABILITIES
SHØULD BE CØMPUTED FØR
 INPUT M=0 IF CØMPUTATIØN NØT DESIRED
? ?3

CMP: MATRIX ØF TRANSITIØN
PRØBABILITIES FØR M STEPS,M=        3
PRØBABILITY ØF GØING FRØM STATE TØ STATE

STATE    MRØ    PASØRD REJCUS BACKØR FRSPAS SUBPAS
 MRØ     1.000  0.000  0.000  0.000  0.000  0.000
 PASØRD  0.000  1.000  0.000  0.000  0.000  0.000
 REJCUS  0.000  0.000  1.000  0.000  0.000  0.000
 BACKØR  0.000  0.000  0.000  1.000  0.000  0.000
 FRSPAS   .563   .040   .079   .309  0.000   .010
 SUBPAS   .077   .115   .282   .513  0.000   .012

AM, RØW VECTØR FØR STATE SPACE M STEPS LATER,M=        3
GIVEN INITIAL STATE A

STATE    MRØ    PASØRD REJCUS BACKØR FRSPAS SUBPAS
          .466   .055   .119   .349  0.000   .010
```

I 3 Model B of Requisition Processing System

```
MARK1      14:20   MØN.---07/08/68

IN MARK2
IN .FIRST
IN MARK3
IN .FIRST

SIMULATIØN WITH
MARKØV TRANSITIØN MATRIX MØDEL ØF
A REQUISITIØN PRØCESSING SYSTEM
BY IRWIN GØØDMAN
INPUT, C, TRANSITIØN MATRIX IN CANØNICAL FØRM
IN LINES 1420 THRU 1800 BY RØWS
PRECEDE C MATRIX DATA WITH NAMES ØF STATES
RIGHT JUSTIFY BETWEEN CØMMAS WITH SPACES ADDED FØR
7 CHARACTER FØRMAT.
EXAMPLE;   HØME,    BAR, CØRNR2,ETC. FØLLØWED BY DATA
INPUT QTY ØF ABSØRBING STATES=IABSØR
? ?1
INPUT QTY ØF TRANSIENT STATES=ITRANS
? ?2
QTY ØF RØWS IN MATRIX C =        3
QTY ØF CØLUMNS IN MATRIX C =        3


TRANSITIØN MATRIX, C, IN CANØNICAL FØRM

STATE   CØMPLT FRSPAS SUBPAS
 CØMPLT  1.000  0.000  0.000
 FRSPAS   .820  0.000   .180
 SUBPAS   .770  0.000   .230


PARTITIØN TRANSITIØN MATRIX (CANØNICAL FØRM)
INTØ FØLLØWING MATRICIES:
CI: IDENTITY MATRIX

STATE   CØMPLT
 CØMPLT  1.000


CR: MATRIX ØF TRANSIENT TØ ABSØRBING PRØBABILITIES-
PRØBABILITY ØF GØING FRØM TRANSIENT STATE
TØ ABSØRBING STATE

STATE   CØMPLT
 FRSPAS   .820
 SUBPAS   .770
```

```
CQ: MATRIX ØF TRANSIENT TØ TRANSIENT PRØBABILITIES-
PRØBABILITY ØF GØING FRØM TRANSIENT STATE
TØ TRANSIENT STATE

STATE   FRSPAS SUBPAS
 FRSPAS 0.000   .180
 SUBPAS 0.000   .230


CØMPUTATIØN ØF THE FUNDAMENTAL MATRIX
FN: FUNDAMENTAL MATRIX-
EACH ELEMENT IS THE EXPECTED NUMBER ØF TIMES
IN STATE J(CØLUMN) BEFØRE BEING ABSØRBED
GIVEN THAT THE PRESENT STATE IS I(RØW)

STATE   FRSPAS SUBPAS
 FRSPAS 1.000   .234
 SUBPAS 0.000  1.299


CØMPUTATIØN ØF THE MATRIX ØF ABSØRPTIØN TIMES
T: MATRIX ØF ABSØRPTIØN TIMES-
EACH ELEMENT IS THE MEAN TIME TØ ABSØRPTIØN
(NUMBER ØF STATES PASSED THRØUGH, INCLUDING FINAL STATE
AND NØT INCLUDING INITIAL STATE, INØRDER TØ BE ABSØRBED)

STATE
 FRSPAS  1.234
 SUBPAS  1.299


CØMPUTATIØN ØF THE MATRIX ØF ABSØRPTIØN PRØBABILITIES
U, MATRIX ØF ABSØRPTIØN PRØBABILITIES
PRØBABILITY ØF BEING ABSØRBED,
GIVEN IT WAS INITIALLY IN A TRANSIENT STATE

STATE   CØMPLT
 FRSPAS  1.000
 SUBPAS  1.000


CØMPUTATIØN ØF TRANSIENT PRØBABILITY MATRIX FØR M STEPS
INPUT NUMBER ØF STEPS, M,
THAT MATRIX ØF TRANSITIØN PRØBABILITIES
SHØULD BE CØMPUTED FØR
 INPUT M=0 IF CØMPUTATIØN NØT DESIRED
? ?2


CMP: MATRIX ØF TRANSITIØN
PRØBABILITIES FØR M STEPS,M=        2
PRØBABILITY ØF GØING FRØM STATE TØ STATE

STATE   CØMPLT FRSPAS SUBPAS
 CØMPLT  1.000  0.000  0.000
 FRSPAS   .959  0.000   .041
 SUBPAS   .947  0.000   .053
```

```
COMPUTATION OF STATE SPACE, M STEPS LATER,
GIVEN INITIAL STATE
INPUT ISAMEA=0 IF SAME ROW VECTOR TO BE USED
INPUT ISAMEA=1 FOR NEW A(K) VECTOR INPUT.
? ?1
INPUT INITIAL STATE AS A ROW VECTOR IN FOLLOWING FORM
K=        3
INPUT K=0 IF COMPUTATION NOT DESIRED
INPUT K,A(1),A(2),A(3),....,A(K)
? ?3,0,.80,.20


AM, ROW VECTOR FOR STATE SPACE M STEPS LATER,M=        2
GIVEN INITIAL STATE A

STATE    COMPLT FRSPAS SUBPAS
           .956  0.000   .044


COMPUTATION OF TRANSIENT PROBABILITY MATRIX FOR M STEPS
INPUT NUMBER OF STEPS, M,
THAT MATRIX OF TRANSITION PROBABILITIES
SHOULD BE COMPUTED FOR
 INPUT M=0 IF COMPUTATION NOT DESIRED
? ?3


CMP: MATRIX OF TRANSITION
PROBABILITIES FOR M STEPS,M=        3
PROBABILITY OF GOING FROM STATE TO STATE

STATE   COMPLT FRSPAS SUBPAS
 COMPLT  1.000  0.000  0.000
 FRSPAS   .990  0.000   .010
 SUBPAS   .988  0.000   .012


COMPUTATION OF STATE SPACE, M STEPS LATER,
GIVEN INITIAL STATE
INPUT ISAMEA=0 IF SAME ROW VECTOR TO BE USED
INPUT ISAMEA=1 FOR NEW A(K) VECTOR INPUT.
? ?0


AM, ROW VECTOR FOR STATE SPACE M STEPS LATER,M=        3
GIVEN INITIAL STATE A

STATE    COMPLT FRSPAS SUBPAS
           .990  0.000   .010
```

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A cursory review of the literature relating to the application of the Markov Transition Probability Matrix for the evaluation and analysis of problems was accomplished. A FORTRAN IV computer time-sharing program, based upon the mathematics of Markov Transition Matrices, has been developed and documented. The program was initially developed with data based upon a classical random walk problem involving a drunk meandering from corner to corner between his home and a bar. The resulting Markov Model has been applied to a requisitioning system, an essentially equivalent problem. Some analysis results are presented following the application of the computer program to a requisitioning system. The computer program has been written generally enough for application to such other diverse problem areas as charge accounts, tank battles, and reliability and maintainability.

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